R.1 Exponents and Order of Operations

Exponents

There are many mathematical situations in which we multiply a number by itself repeatedly. Writing such expressions using *exponents* (or *powers*) provides a shorthand method for representing this repeated multiplication of the same factor:

$$\underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{\textbf{\textit{4 factors of 2}}} = 2^4 \quad \leftarrow \text{exponent}; \; 2 = \text{base}$$

The expression 2^4 is read "2 to the fourth power" or simply "2 to the fourth."

To evaluate 2^4, we multiply 4 factors of 2:

$$\begin{aligned} 2^4 &= \underbrace{2 \cdot 2} \cdot 2 \cdot 2 \\ &= \underbrace{4 \cdot 2} \cdot 2 \\ &= \underbrace{8 \cdot 2} \\ &= 16 \end{aligned}$$

So $2^4 = 16$.

Sometimes we prefer to shorten expressions by using exponents. For instance,

$$\underbrace{3 \cdot 3}_{\textbf{2 factors of 3}} \cdot \underbrace{4 \cdot 4 \cdot 4}_{\textbf{3 factors of 4}} = 3^2 \cdot 4^3$$

EXAMPLE 1

Rewrite $6 \cdot 6 \cdot 6$ using exponents.

Solution $\underbrace{6 \cdot 6 \cdot 6}_{\textbf{3 factors of 6}} = 6^3$

PRACTICE 1

Write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as a power of 2.

EXAMPLE 2

Calculate: $4^3 \cdot 5^3$

Solution

$$\begin{aligned} 4^3 \cdot 5^3 &= (4 \cdot 4 \cdot 4) \cdot (5 \cdot 5 \cdot 5) \\ &= 64 \cdot 125 \\ &= 8000 \end{aligned}$$

PRACTICE 2

Compute: $7^2 \cdot 2^4$

It is especially easy to compute powers of 10:

$$10^2 = \underbrace{10 \cdot 10}_{\textbf{2 factors}} = \underbrace{100}_{\textbf{2 zeros}}$$

$$10^3 = \underbrace{10 \cdot 10 \cdot 10}_{\textbf{3 factors}} = \underbrace{1000}_{\textbf{3 zeros}}$$

and so on.

EXAMPLE 3

The distance from the Sun to the star Alpha-one Crucis is about 1,000,000,000,000,000 miles. Express this number as a power of 10.

Solution $\underbrace{1{,}000{,}000{,}000{,}000{,}000}_{\text{15 zeros}} = 10^{15}$ (miles)

PRACTICE 3

In 1850, the world population was approximately 1,000,000,000. Represent this number as a power of 10.
(Source: World Almanac and Book of Facts 2000)

Order of Operations

Some mathematical expressions involve more than one mathematical operation. For instance, consider the expression $5 + 3 \cdot 2$. This expression seems to have two different values, depending on the order in which we perform the given operations.

Add first	**Multiply first**
$\underbrace{5 + 3} \cdot 2$	$5 + \underbrace{3 \cdot 2}$
$= \underbrace{8 \cdot 2}$	$= \underbrace{5 + 6}$
$= 16$	$= 11$

How do we know which operation to carry out first? By consensus we agree to follow the rule called the **order of operations** so that everyone always gets the same value as the answer.

Order of Operations Rule

To evaluate mathematical expressions, carry out the operations *in the following order:*

1. First, perform the operations within any grouping symbols, such as parentheses ( ) or brackets [ ].
2. Then raise any number to its power.
3. Next, perform all multiplications and divisions as they appear from left to right.
4. Finally, do all additions and subtractions as they appear from left to right.

Applying the order of operations rule to the previous example gives us the following result:

$$5 + \underbrace{3 \cdot 2} \quad \textbf{Multiply first.}$$
$$= \underbrace{5 + 6} \quad \textbf{Then add.}$$
$$= 11$$

So 11 is the correct answer.

EXAMPLE 4

Simplify: $20 - 18 \div 9 \cdot 8$

Solution The order of operations rule gives us

$$\begin{aligned} 20 - 18 \div 9 \cdot 8 &= 20 - 2 \cdot 8 && \textbf{Divide first.} \\ &= 20 - 16 && \textbf{Then multiply.} \\ &= 4 && \textbf{Finally subtract.} \end{aligned}$$

PRACTICE 4

Evaluate: $8 \div 2 + 4 \cdot 3$

EXAMPLE 5

Find the value of $3 + 2 \cdot (8 + 3^2)$.

Solution

$$\begin{aligned} 3 + 2 \cdot (8 + 3^2) &= 3 + 2 \cdot \underbrace{(8 + 9)} && \textbf{Perform operations in parentheses: square the 3.} \\ &= 3 + \underbrace{2 \cdot 17} && \textbf{Add 8 and 9.} \\ &= \underbrace{3 + 34} && \textbf{Multiply 2 and 17.} \\ &= 37 && \textbf{Add 3 and 34.} \end{aligned}$$

PRACTICE 5

Simplify: $(4 + 1)^2 - 4 \cdot 6$

Tip When a division problem is written as a fraction, parentheses are understood to be around both the numerator and the denominator. For instance,

$$\frac{10 - 2}{3 - 1} \quad \text{means} \quad (10 - 2) \div (3 - 1)$$

EXAMPLE 6

A student's test scores in a class were 85, 94, 93, 86, and 92. If the average of these scores was 90 or above, she earned an A. Did the student earn an A?

Solution

$$\begin{aligned} \text{Average} &= \frac{\text{The sum of the scores}}{\text{The number of scores}} \\ &= \frac{85 + 94 + 93 + 86 + 92}{5} \\ &= (85 + 94 + 93 + 86 + 92) \div 5 \\ &= 450 \div 5 \\ &= 90 \end{aligned}$$

Therefore, the student did earn an A.

PRACTICE 6

Last year, a tenant's average monthly electricity bill was \$110. This year, these bills amounted to \$84, \$85, \$88, \$92, \$80, \$96, \$150, \$175, \$100, \$95, \$75, and \$80. On average, were the monthly bills higher this year?