

R.2

Factors, Primes, and Least Common Multiple

Recall that in a multiplication problem involving two or more whole numbers, the whole numbers that are multiplied are called **factors**. For example, since $2 \cdot 4 = 8$, we say that 2 is a factor of 8, or that 8 is divisible by 2. The factors of 8 are 1, 2, 4, and 8.

Now let's discuss the difference between prime numbers and composite numbers. A **prime number** is a whole number that has exactly two factors, namely, itself and 1. A **composite number** is a whole number that has more than two factors. For instance, 5 is prime because its only factors are 1 and 5. However, 9 is a composite because it has more than two factors, namely, 1, 3, and 9. Note that both 0 and 1 are considered neither prime nor composite.

Every composite number can be written as the product of prime factors, called its **prime factorization**. For instance, the prime factorization of 12 is $2 \cdot 2 \cdot 3$, or $2^2 \cdot 3$. This factorization is unique.

The **multiples** of a number are the products of that number and the whole numbers. For instance some of the multiples of 5 are

$$\begin{array}{cccc} 0 & 5 & 10 & 15 \\ \hline 0 \times 5 & 1 \times 5 & 2 \times 5 & 3 \times 5 \end{array}$$

A number that is a multiple of two or more numbers is called a **common multiple** of these numbers. Some of the common multiples of 6 and 8 are 24, 48, and 72.

The **least common multiple (LCM)** of two or more numbers is the smallest nonzero number that is a multiple of each number. For example, the LCM of 6 and 8 is 24.

A good way to find the LCM involves prime factorization.

EXAMPLE 1

Find the LCM of 8 and 12.

Solution We first find the prime factorization of each number.

$$8 = 2 \cdot 2 \cdot 2 = 2^3 \quad 12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$$

Since 2 appears *three* times in the factorization of 8 and *twice* in the factorization of 12, it must be included three times in forming the least common multiple. So we use 2^3 (the highest power of 2 in the factorizations). The factor of 3 must also be included. Therefore, the LCM of 8 and 12 is

$$\text{LCM} = 2^3 \cdot 3 = 8 \cdot 3 = 24$$

PRACTICE 1

What is the LCM of 10 and 25?