# 1.5 Properties of Real Numbers

**OBJECTIVES**

- To use the commutative properties
- To use the associative properties
- To use the identity properties
- To use the inverse properties
- To use the distributive property
- To solve applied problems involving the properties of real numbers

In this section, we focus on some of the key properties of real numbers—the commutative properties, the associative properties, the identity properties, the inverse properties, and the distributive property. These properties are very important to the study of algebra, for they underlie algebraic procedures and, in particular, they help us to simplify complicated expressions.

You may wish to review the multiplicative property of 0 and the material about division involving 0 in Section 1.4.

## The Commutative Properties

Let's begin by considering the two commutative properties: the *commutative property of addition* and the *commutative property of multiplication*. The commutative property of addition states that we get the same sum when we add two numbers regardless of order. For instance, the sums $3 + 5$ and $5 + 3$ give the same result, 8, even though the order in which the 3 and 5 are added differs. So we write: $3 + 5 = 5 + 3$.

**Commutative Property of Addition**

For any real numbers $a$ and $b$,

$$a + b = b + a.$$

Note that in this statement of the commutative property, as well as in the other properties of real numbers, symbols are used to represent real numbers: The letter $a$ represents one number and the letter $b$ represents another number. In algebra, a *variable* is a quantity that is unknown, that is, one that can change or vary in value.

**EXAMPLE 1**

Rewrite each expression using the commutative property of addition.

**a.** $7 + (-2)$ **b.** $2x + y$

**Solution**

**a.** $7 + (-2) = (-2) + 7$, or $-2 + 7$

**b.** $2x + y = y + 2x$

**PRACTICE 1**

Use the commutative property of addition to rewrite each expression.

**a.** $-3 + 5$

**b.** $b + 3a$

The commutative property of multiplication says that we get the same product when we multiply two numbers in any order. For example, the products $3 \cdot 5$ and $5 \cdot 3$ are the same, 15, even though the order in which the 3 and 5 are multiplied differs. So we write: $3 \cdot 5 = 5 \cdot 3$.

**Commutative Property of Multiplication**

For any real numbers $a$ and $b$,

$$a \cdot b = b \cdot a.$$

Note that when writing an expression involving a product, we can either omit the multiplication symbol or use parentheses. For instance, we can write $a \cdot b$ as either $ab$ or $(a)(b)$.

**EXAMPLE 2**

Rewrite each expression using the commutative property of multiplication.

**a.** $(-5)(-3)$ **b.** $x(-7)$

**Solution**

**a.** $(-5)(-3) = (-3)(-5)$ **b.** $x(-7) = (-7)x$, or $-7x$

**PRACTICE 2**

Use the commutative property of multiplication to rewrite each expression.

**a.** $(-8)(2)$

**b.** $-4n$

# The Associative Properties

Next we consider the two associative properties: the *associative property of addition* and the *associative property of multiplication.* The first of these properties says that if we regroup the three numbers, we get the same sum. For instance, the result of adding 2, 3, and 6 is the same, 11, whether we add 2 and 3 first and then add 6 to this sum or whether we add 3 and 6 first and then add 2 to this sum. So we write: $(2 + 3) + 6 = 2 + (3 + 6)$.

**Associative Property of Addition**

For any real numbers $a$, $b$, and $c$,

$$(a + b) + c = a + (b + c).$$

**EXAMPLE 3**

Rewrite each expression using the associative property of addition.

**a.** $[(-4) + (-3)] + 6$ **b.** $(2p + q) + r$

**Solution**

**a.** $[(-4) + (-3)] + 6 = (-4) + [(-3) + 6]$

**b.** $(2p + q) + r = 2p + (q + r)$

**PRACTICE 3**

Use the associative property of addition to rewrite each expression.

**a.** $[8 + (-1)] + 2$

**b.** $(x + 3y) + z$