Math 215
Study Sheet for Test 2

For all the GCF/LCM problems, you are expected to use (and to show) the methods we learned in class.

Assume all letters represent counting numbers.

1. Indicate whether each statement is true or false:

a) 6 is a multiple of 12.
b) Any multiple of 9 will also be a multiple of 12.
c) If a number is made up of 18's, it is also made up of 4's.
d) Any counting number that is not prime is composite.
e) 93,198,624 is not divisible by 6.
f) 1 and m are relatively prime.
g) If two different numbers are prime, then they must be relatively prime.
h) Any counting number which is not prime must be composite.
i) The GCF of $6x$ and $7x$ is x.
j) The LCM of two different prime numbers is their product.
k) The LCM of n and 1 is 1.
a) If LCM$(x, y) = 16$, then x is a multiple of 16.
b) If a and b are relatively prime, then GCF$(a, b) = b$.
c) If GCF$(m, n) = 6$, then n is a factor of 6.
d) If a is a factor of b and $a \neq b$, LCM$(a, b) = b$.
e) GCF$(a, 1) = a$.
f) If a and b are both multiples of 3, they are relatively prime.
g) Two consecutive odd numbers are always relatively prime.
h) Two consecutive even numbers are always relatively prime.

2. List all the factors of 48.

3. Why is the sum of the numbers in a row of Pascal's Triangle exactly twice the sum of the numbers in the previous row? Give a clear and convincing explanation.

4. The numbers 1, 4, 10, 20, 35 are what kinds of numbers?

5. Show where the Fibonacci numbers can be found in Pascal's Triangle (besides in the counting number "column").

6. Systematically list all the factors of 6^5 in their prime factorizations.

7. Systematically list all the factors of $2^3 \cdot 3$ in their prime factorizations.

8. Write any 10 factors of $2^4 \cdot 5^9 \cdot 13$, in their prime factorizations.

9. Name two numbers, both above 20, that are relatively prime.

10. Explain: any multiple of 5 that is greater than 5 must be composite.

11. What is necessary to show that a number is composite?

12. Find the smallest composite number that is not divisible by any counting number between 1 and 50.

13. Name two different numbers, both between 20 and 300, that have a GCF of 15.

14. Name two different numbers that have an LCM of 30.

15. Suppose Chris and Jean have different work schedules. Chris has every 4th day off, and Jean has 3rd day off. (If they both worked March 31st, Chris will be off on April 4th and Jean will be off April 3rd.) Which of them will be off on April 30th? What is the first day in April they will both have off?

16. Find, using the technique we used in class, LCM(24, 45).

17. Find, using the technique we used in class, GCF(350, 630).

18. Name four pairs of numbers that have an LCM of 100. (Your answers should be 6 different numbers.)

19. What is the name of the "filter" which can be used to find prime numbers in the first n counting numbers? Briefly explain how one uses this "filter."

20. When using the "filter" described in problem 14, what would be the first number one would cross out after circling 73? (73 is a prime numbers.) Write the answer in its prime factorization.

21. What pairs of numbers have a GCF of 5 and an LCM of 125? (List all of the possibilities.)

22. If GCF $(x, 42) = 6$ and LCM $(x, 42) = 252$, find x.

23. If GCF $(x, 85) = 5$ and LCM $(x, 85) = 850$, find x.

24. If GCF $(x, 18) = 2$ and LCM $(x, 18) = 504$, find x.

25. If GCF $(x, 48) = 16$ and LCM $(x, 48) = 192$, find x.

26. If GCF $(x, 162) = 6$ and LCM $(x, 162) = 324$, find x.

27. If GCF $(x, 72) = 8$ and LCM $(x, 72) = 504$, find x.

28. If GCF $(x, 32) = 16$ and LCM $(x, 32) = 96$, find x.

29. If GCF $(x, 48) = 3$ and LCM $(x, 48) = 432$, find x.

30. Name two numbers between 200 and 1000 that have a GCF of 9.

31. Evaluate:

a) GCF $(x, 3x) =$

b) LCM $(b, 4b) =$

c) GCF $(a, 4a^2) =$

d) LCM $(n, 1) =$

e) LCM (p, q) if p and q are different prime numbers

f) LCM ($2p$, $3q$) if p and q are different prime numbers

g) LCM($6p$, $10q$) if p and q are different prime numbers

h) GCF(1, a, $3a$)

i) LCM(1, a, $3a$)

32. If the GCF of two numbers is 8 and their LCM is 320, what could the numbers be? (List all the possibilities.)

33. Name three pairs of numbers x and y, so that $x \neq y$, neither x nor y is a multiple of 7, and GCF(x, y) = 4.

34. Name three pairs of different odd numbers that have a GCF of 15.

35. A student claims that all multiples of 3 are composite. Is he correct? Why or why not?

36. Find two numbers that have a GCF of 20 and an LCM of 280. Find three pairs (6 different numbers, if possible).

37. Find two numbers m and n so that m and n are both greater than 100, $m \neq n$, and GCF(m, n) = 21.

38. Find 4 different pairs of numbers x and y, so that $x \neq y$ and LCM(x, y) = 300. (Your answers should be 8 different numbers.)

39. Find all possible pairs of numbers that have an LCM of 121.

40. Find a pair of numbers that has a GCF of 12 and an LCM of 420.

41. Use the rules for divisibility to see if 487,342 is divisible by 3, 4, 6, 8, and 9. (You need to show how you are using each rule, but you do not need to explain why each rule works the way it does.)

42. Briefly, but clearly, explain why the divisibility rule for 8 works.

43. Briefly, but clearly, explain why the divisibility rule for 3 works.

44. Name 4 natural objects that contain Fibonacci numbers.

45. Name the first 20 Fibonacci numbers.

46. Name any 3 mathematical facts (from the approved list) about Fibonacci numbers.

47. Write the first 10 rows of Pascal's Triangle.

48. Briefly describe 6 characteristics of or patterns in Pascal's Triangle.

49. Find the probability of getting no more than 3 heads when tossing a coin 8 times.

50. Find the probability of getting exactly 7 heads when tossing a coin 8 times.

51. Find the probability of getting an even number of heads when tossing a coin 8 times.

52. Find the probability of getting at least 6 heads when tossing a coin 8 times.

53. Rewrite this statement in at least 5 different ways: <u>5 is a factor of *m*</u>. (Each of your statements needs to be mathematically equivalent to the original one.)

54. In the problem $24 \div 8 = 3$, the 24 is referred to as the ___________________________, the 8 is referred to as the ________________________, and the 3 is referred to as the ________________________________.

55. In the problem $3 \times 17 = 51$, the 3 is referred to as the ___________________________, the 17 is referred to as the ________________________, and the 51 is referred to as the _______________________________.

56. Multiply, using the partial products method:

a) 24×56
b) 38×14
c) 64×22

57. Indicate the problems which have the same answer as this problem: $64 \times m$

a) $32 \times \frac{1}{2} m$
b) $60 \times (m + 4)$
c) $128 \times \frac{1}{2} m$
d) $62 \times 2m$
e) $8 \times (8m)$

58. Indicate the problems which have the same answer as this problem: $24 \times 42{,}860$

a) $20 \times 42{,}864$
b) $48 \times 21{,}430$
c) 240×4286
d) 2.4×4286

59. Use the partial quotients method to divide:

a) $5796 \div 16$
b) $5475 \div 12$
c) $6359 \div 18$
d) $3608 \div 16$

60. Divide $567 \div 8$ by decomposing the dividend into 400 and 167.

61. Divide $80 \div 12$ by decomposing the dividend into 40 and 40.

62. Divide $80 \div 12$ by decomposing the dividend into 50 and 30.