## CORRECTION TO TRANSLATION GROUPOIDS AND ORBIFOLD COHOMOLOGY

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This note concerns an error in the proof of Lemma 8.1 of the paper *Translation Groupoids and Orbifold Cohomology*, Canadian J. Math Vol 62 (3), pp 614-645 (2010). This was pointed out to the authors by Li Du of the Georg-August-Universität at Gottingen, who also suggested the outline for the following corrected proof.

The lemma in question reads:

**Lemma 8.1.** The class of essential equivalences between Lie groupoids satisfies the 3-for-2 property, i.e., if we have homomorphisms  $\mathcal{G} \xrightarrow{\varphi} \mathcal{K} \xrightarrow{\psi} \mathcal{H}$  such that two out of  $\{\varphi, \psi, \varphi \circ \psi\}$  are essential equivalences, then so is the third.

The given proof of this lemma is incorrect in the case where  $\psi \circ \varphi$  and  $\psi$  are essentially equivalences. There it is stated:

It is a standard property of fibre products that if any two out of (A), (B), and the whole square are fibre products, so is the third.

This is incorrect in general; in particular, when  $\varphi$  and  $\psi \circ \varphi$  are merely fully faithful it is not necessary that  $\psi$  is also, and counter-examples can be created. Below is a corrected proof of the case in question.

*Proof.* We consider the case where  $\varphi$  and  $\psi \circ \varphi$  are essential equivalences. Since  $\psi \circ \varphi$  is essentially surjective, the map  $G_0 \times_{H_0} H_1 \to H_0$  is a surjective submersion. This map factors as the top arrow in the following diagram,

$$\begin{array}{cccc} G_0 \times_{H_0} H_1 \longrightarrow K_0 \times_{H_0} H_1 \longrightarrow H_1 \stackrel{t}{\longrightarrow} H_0 \\ & & & & \downarrow & & \downarrow s \\ G_0 \stackrel{\varphi_0}{\longrightarrow} K_0 \stackrel{\psi_0}{\longrightarrow} H_0 \end{array}$$

and we see that this implies that the composite of the last two maps,  $K_0 \times_{H_0} H_1 \rightarrow H_0$  is a surjective submersion.

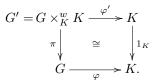
Next we consider the following diagram

$$\begin{array}{c|c} G_1 & \xrightarrow{\varphi_1} & K_1 & \xrightarrow{\psi_1} & H_1 \\ (s,t) & & (A) & (s,t) & & (B) & & (s,t) \\ G_0 \times G_0 & \xrightarrow{\varphi_0 \times \varphi_0} & K_0 \times K_0 & \xrightarrow{\psi_0 \times \psi_0} & H_0 \times H_0. \end{array}$$

Since  $\varphi$  and  $\psi \circ \varphi$  are essential equivalences, the left square (A) and the entire rectangle are both pullbacks. We want to show that the right square has to be a pullback as well. As indicated by the discussion above, the fact that  $\varphi$  is essentially

surjective is an important ingredient. In fact, we would like to assume that  $\varphi_0$  is actually surjective.

If  $\varphi_0$  is not surjective, then consider the weak pullback groupoid



Since  $\varphi$  is an essential equivalence, so is  $\varphi'$ . In addition,  $\pi$  is also an essential equivalence, because it is a weak pullback of an identity arrow (which is obviously an essential equivalence).

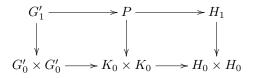
So we replace (A) by a new square (A'), which is again a pullback:

$$\begin{array}{c|c} G_1' & \xrightarrow{\varphi_1'} & K_1 & \xrightarrow{\psi_1} & H_1 \\ (s,t) & & & & & \\ (s,t) & & & & & \\ G_0' \times G_0' & \xrightarrow{(A')} & K_0 \times K_0 & \xrightarrow{(B)} & H_0 \times H_0. \end{array}$$

Furthermore, the entire rectangle is again a pullback: Note that  $\psi \circ \varphi' \cong (\psi \circ \varphi) \circ \pi$ . The latter is an essential equivalence as a composite of essential equivalences and hence so is the former, because it is isomorphic to an essential equivalence. We have also that the map  $\varphi': G'_0 = G_0 \times_{K_0} K_1 \times_{K_0} K_0 \to K_0$ , defined by  $(x, k, t(k)) \mapsto t(k)$ , is surjective since  $\varphi$  is essentially surjective.

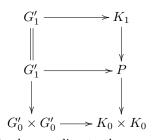
Now consider the pullback

Since the map  $s\pi_1$  is a surjective submersion, this pullback is a smooth manifold, and we get a smooth map  $K_1 \to P = K_0 \times_{H_0,s} H_1 \times_{t,H_0} K_0$ . Next consider the diagram



We know that the right square is a pullback, and therefore the left square is a pullback if and only if the whole rectangle is a pullback. But the whole rectangle is a pullback as we just observed and so the left square is a pullback.

So now consider



The bottom square is a pullback according to the previous argument, and we know that the whole rectangle is a pullback since  $\varphi' : G' \to K$  is fully faithful. Therefore, the top square is also a pullback.

Now the bottom map is a surjective submersion (it is surjective as argued above and it is submersion because the groupoids are étale), and therefore the pullback map  $G'_1 \to P$  is also a surjective submersion. Then looking at the top square, we see that the pullback of the map  $K_1 \to P$  is the identity map, and hence a diffeomorphism. Therefore the original map must have also been a diffeomorphism. So  $K_1 \cong P$  and so the original square (B) is a pullback as required.  $\Box$