Ideas in Practice: Collaborative Problem-Based Learning in Intermediate Algebra

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ABSTRACT: *A key goal in developmental education has been optimizing student success in future college-level classes. This study compared three sections of a problem-based collaborative learning pilot course of Intermediate Algebra to the original course section at a four-year public liberal arts college. The pilot course differed from the original course in three main areas: structure, content, and assessments. Results showed that student performance and satisfaction with the pilot course did not differ significantly from the usual course but that success in College Algebra the following semester was significantly higher among students from the pilot course sections, especially for Native Americans.*

> *The success or failure of developmental mathematics courses impacts the lives of hundreds of thousands of college students.*

There is a growing sense of national urgency around improving the field of developmental mathematics (Cullinane & Treisman, 2010). College educators have long recognized a problem in the plethora of students placed into developmental mathematics courses along with the low success rates of students within those courses. Furthermore, research is showing that even students who do successfully complete their developmental prerequisite courses are not often successful in their first college-level mathematics course, thereby making developmental mathematics "a burial ground for the aspirations of myriad college students trying to improve their lives through education" (Cullinane & Treisman, 2010, p. 2).

Statistics on developmental education are often inconsistent due to different definitions for college-level placement between states, within states, and, sometimes, even within institutions (Attewell, Lavin, Domina, & Levey, 2006). However, even a small sample of the extant data provides ample evidence that the success or failure of developmental mathematics courses impacts the lives of hundreds of thousands of college students nationwide. Only 45% of high school graduates in 2011 met the benchmark for being "college ready" in mathematics ("Condition of College...," 2011). Accordingly, the National Educational Longitudinal Study found that 28% of traditional undergraduates take at least one developmental mathematics course (Attewell et al., 2006). In 2010, 35% of the 47,885 first-time freshmen in the California State University (CSU) system were not college-ready in mathematics ("CSU online database," 2010). Minority students in the CSU system were far more likely to be referred to developmental mathematics, with rates of 35% for Native-American, 64% for African-American, and 49% for Mexican and Latino students compared to 21% for White students.

Developmental course enrollment numbers are even higher for community college students. Analysis of the 2004 data from 53 community colleges found that 59% of students were placed into developmental mathematics courses, with minority students again overrepresented (Bailey, Jeong, & Cho, 2010). Overall, only 33% of these students successfully completed their entire one-, two-, or three-course developmental sequence. Furthermore, only 50% of the developmental mathematics completers—16% of the total number who began in the developmental sequence—passed their first college-level mathematics course. The data from Colorado closely mirror these National trends: Of the almost 7,000 new remedial math students beginning in the Colorado Community College System in Fall 2003, 44% of them successfully completed their developmental mathematics sequence, with 41% of these completers (18% total) passing college-level mathematics during the course of the 4-year period of observation (Nawrocki, Baker, & Corash, 2009). Accordingly, large-scale studies that have examined students on either side of the developmental mathematics cut-off point have found that course completion does not improve success in the first college-level course (Bailey, 2008); that is, taking developmental courses has been only marginally successful at helping students who place into them get their mathematics skills up to college level.

The pilot project described in the present article sought to address this issue by improving the success of developmental mathematics students in college-level courses without negatively impacting their success rate in the developmental course. Specifically, this study investigated the effects of a problem-based collaborative learning intervention in Intermediate Algebra on student performance in their first college-level mathematics course the following semester.

Background

The college at which this study was conducted has a separate department to teach developmental

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mathematics courses termed the Freshman Math Program (FMP). FMP faculty members have a background in mathematics education rather than higher-level mathematics per se:. They are trained as teachers of mathematics rather than doctoral researchers. The FMP is responsible for two college-level courses and two developmental courses—Introduction to Algebra and Intermediate Algebra—that are not offered for college credit but rather as a benefit to students (Duranczyk & Higbee, 2006). In 2008, the college received a Title III Strengthening Institutions Grant from the U.S. Department of Education for the purpose of improving mathematics courses and instruction, especially among underserved populations such as Native-American students. This grant has supported the work to redesign the Intermediate Algebra course described herein.

The primary goal of Intermediate Algebra has been to teach students manipulative algebraic skills for use in future college-level mathematics classes. The course had previously been taught in a lecture format with some reform-based strategies, such as problem applications and regular use of graphing calculators, which have been shown empirically to aid student performance in beginning algebra classes (Martin, 2008). However, the course did not effectively integrate collaborative learning or mathematical writing. Moreover, because the course is a prerequisite for several distinct college-level mathematics classes, the required skill set was broad and vast (Yopp & Rehberger, 2009). The course redesign team thus identified four main areas of concern:

- The course was not rigorous enough both in terms of actual content and in terms of the level of thinking expected of students. The latter was defined as the level of cognitive demand, based on the work of Stein, Smith, Henningsen, and Silver (2000).
- Skills and concepts were taught in an isolated and disconnected manner, which made retention of the learning less likely (Beyer, 1991; Zavarella & Ignash, 2009).
- Students were already using computers (Math XL; see method) to supplement the class and textbook material, but computer use alone has not been shown to boost mathematics performance in developmental classes (Spradlin & Ackerman, 2010).
- The course did not effectively boost students' self-efficacy in learning mathematics, a factor that may be especially germane for Native-American students (House, 2009).

Prior to designing the pilot (revised) Intermediate Algebra course, the FMP faculty conducted an extensive literature search on best practices in teaching mathematics in general and developmental mathematics more specifically. Armington (2003) helped sharpen the key concepts that the redesign team chose to pursue; notably, capstone problems, a hybrid model of lecture and computer work, collaborative activities, and writing activities. Bottage, Heinrich, Chan, and Serlin (2001) provided samples of these capstone problems and collaborative activities, whereas Goolsby (1988) and Weens (1998) highlighted pedagogical issues regarding the link between homework and mathematics achievement. Based on this work, the team identified three general goals and several supporting strategies to be implemented in the redesign in order to address the four main concerns, as shown in Table 1. The key components of the intervention included collaborative, problem-based learning along with a capstone project. The study's main hypothesis was that the new curriculum would better prepare students for their future college-level mathematics classes, arguably the chief purpose of developmental courses.

Accordingly, the current research aimed to answer the following four questions: (a) would the students persist in the redesigned (and likely more challenging) pilot course; (b) how would these students perform compared with the original (nonredesigned) class; (c) what would the long-term effects of the redesigned curriculum be (i.e., whether these students would perform better in their next college-level mathematics class); and (d) how would student satisfaction in the redesigned course compare to the original course.

Table 1

Goals and Supporting Strategies of the Intermediate Algebra Redesign Team

Goal	Supporting strategy
Give mathematics more context and meaning with connections to previous topics and other areas	• Capstone Problem • Applications (e.g., how much energy a green vs. regular light bulb emits, U.S. vs. Canada gas prices)
Engage students in high cognitive demand tasks to prepare them for future mathematics classes	• Activity-based and collaborative learning • Multiple types of assessment including writing
Bolster students' mathematical self-efficacy	• Hybrid lecture/lab format • Frequent feedback with opportunities to correct

Methods

Students enter the college under study with two primary standardized scores: the ACT and the Colorado Commission on Higher Education (CCHE) index. The ACT test assesses high school students' general educational development and their ability to complete college-level work. The multiple-choice tests cover four skill areas: English, mathematics, reading, and science ("The ACT," 2010). The average mathematics score for ACT test takers in 2009 was 20.5 in the state and 21.0 nationally ("ACT Profile Report–State," 2010).

The state higher education commission also uses the CCHE score to indicate college readiness. For new applicants to a college, the CCHE index score is calculated using a combination of a student's high school GPA combined with ACT or SAT score. This index is used by colleges to make admission decisions, with minimum scores in Fall 2009 ranging from 76 to 110; our college is moderately selective with a CCHE index score minimum of 92 ("CCHE Admission Eligibility Index," 2011).

The determination of whether a student takes a developmental or college-level mathematics class when they enter college is stipulated by state law. For mathematics, a student must have an ACT Math score of 19 or above to take a college-level mathematics course, with SAT and Accuplacer equivalents allowed. The Accuplacer is a standardized test that assesses a student's level of academic readiness in mathematics, reading, and English (available at http://www.collegeboard.com/student/testing/accuplacer/) and has shown strong predictive validity for developmental mathematics courses (James, 2006). Institutions set their own placements on either side of this cut-off (e.g., Donovan & Wheland, 2008). The college under study places students with ACT Math scores of 17-18 into Intermediate Algebra, whereas students with scores below 17 must first take Introduction to Algebra.

Setting

This study was conducted at a four-year public liberal arts college with a student enrollment of approximately 3,700 students, about 20% of whom are Native

American. Despite increased admission standards at the college since 2004, 34% of the 2008 freshman class placed below college level in mathematics, with 30% of these students being Native American.

Participants

Demographics of the different course sections were generally comparable across the board, with similar gender and ethnic breakdowns. However, there were significantly more freshman in the pilot classes versus the original class (69.6% vs. 44.8%, $\chi^2(2, N = 108) = 6.49, p = .04$). Although the percentage of Native-American students was higher in the original class versus the pilot classes (41.4% vs. 32.9%), this difference was not statistically significant ($\chi^2(2, N = 106) = .67, p = .72$). Most importantly, however, the two key predictors of mathematics performance described previously—Math ACT score and the CCHE index—were basically identical between the two groups (see Table 2).

Design and Procedure

Study design. The Intermediate Algebra course had four different sections of approximately 30 students in each section. The same instructor (the first author, LG) taught one original course section and three redesigned pilot sections in the same college semester (Fall 2009), which allowed for comparison across sections without considerable instructor variance.

Course design. The pilot course differed from the original course in three main areas: structure, content, and assessments.

Structure. Generally, best practices in developmental education include varying instructional methods in order to accommodate different learning styles and providing immediate feedback, noting student difficulties along with what can be done to improve understanding and performance (Silverman & Casazza, 1999). The pilot course thus utilized a hybrid lecture/lab structure. In a 3-week rotation, six of the seven class periods were devoted to "classroom time," which included direct instruction, activity-based group work, and class discussion. The group work was designed to support students in thinking through new ideas and processes with ample and consistent feedback. For instance, students were first given direct instruction on the basics of solving two-step equations. Students were then given the task of solving equations with radicals and powers without further instruction, so that they had to use what they knew about equations, powers, and roots to perform a task they had never seen before. The instructor employed a scaffolding approach (van de Pol, Volman, & Beishuizen, 2010), providing just enough help for the students to proceed but rely primarily on the strength of their own ideas.

The seventh class period in the 3-week rotation was devoted to student work using Math XL on computers. Math XL is an online tutorial and homework system that accompanied the course textbook (Akst & Bragg, 2008), and enabled students to receive individualized instruction from the teacher as needed while working at their own pace, something that may be especially valuable for Native-American students (Guillory, 2009). In the original course, students also used Math XL, but they did so only at home (on their own), rather than in the pilot classroom with assistance and feedback readily available (see Spradlin & Ackerman, 2010, for a discussion of using computers to boost learning in developmental mathematics). Although there were minimum deadlines for completion of work, students could also work ahead as appropriate and thereby build self-efficacy (Wadsworth, Husman, Duggan, & Pennington, 2007).

Content. Several changes were made to the content covered in the course, though the major content-based changes were to the order/schedule and the use of a capstone problem rather than to the material taught per se. A capstone problem is a way to motivate learning (Armington, 2003); it is introduced early in the semester and utilizes many of the skills developed throughout the course. Students progress in their work on the problem as they learn the requisite skills. The capstone problem used in the pilot course entailed adding fuel additive to a car to determine if gas efficiency increased. The solution involved writing equations, unit analysis, revising equations, determining inputs and outputs from a table, comparing linear functions, and solving a system of equations. Although this capstone did not cover all the material taught in the course (most notably exponents, factoring, and quadratic functions), it furnished an organizational structure that integrated many interrelated course skills and concepts.

The concern about the disjointed nature of the content of the original Intermediate Algebra course precipitated major changes to the order in which topics were taught. The order of major topics for the two courses is shown comparatively in Table 3 (p. 30). The schedule change served several purposes. First, rules and procedures with expressions were grouped together so that students were better able to grasp the importance of these basic skills when the content progressed to functions. Second, examining linear and quadratic functions together allowed students to learn these concepts

Table 2

Student Characteristics by Section (Pilot vs. Original) in Fall 2009

Characteristic	Section	*N*	% or Mean (*SD*)
Gender	Original	29	51.7% female
	Pilot	79	59.5% female
School status	Original	29	44.8% freshman; 34.5% continuing; 13.8% transfer
	Pilot	79	69.6% freshman; 20.3% continuing; 8.9% transfer
Ethnicity	Original	29	55.2% White; 41.4% Native American; 3.4% Hispanic
	Pilot	77	59.5% White; 32.9% Native American; 3.8% Hispanic
Math ACT score	Original	22	17.18 (1.84)
	Pilot	61	17.31 (1.46)
CCHE Index[a]	Original	23	95.04 (8.20)
	Pilot	67	94.49 (9.58)

[a] CCHE Index is a composite score assigned by the Colorado Commission on Higher Education to indicate college readiness.

with both examples and counter-examples. For instance, as students learned about intercepts (i.e., horizontal and vertical), they could compute them in both linear and quadratic functions, but they could also determine that some functions did not have a horizontal intercept. Finally, the comparison of slope in linear functions to the rate of change in quadratic functions emphasized the distinction between constant and variable rates of change.

Assessment. The pilot course retained the basic forms of assessments used in the original course: exams, written and online homework, portfolio, and writing. There were common exam questions asked on both sets of exams (pilot and original), which allowed for direct comparison between the two groups of students. However, because the pilot course covered additional content at a more abstract level, the pilot tests also included several more challenging questions (see Table 1, p. 27, and Table 3, p. 30, for more detail).

There was no difference in the homework or portfolio assignments other than that necessitated by the change in schedule and minor content changes. There was also no difference in the journal component, which involved creating a study schedule, reading about mathematics anxiety, and writing about changes students wanted to make after midterm grades were posted. However, the problem-solving writing component was markedly different between the two sections. In the original course, students solved a problem from their textbook and submitted it for grading at various points throughout the semester. In the pilot course, students were explicitly taught how to show their work in a column or paragraph format as well as how to describe their mathematical steps in writing. Further, the type of problems selected required students to discern what information was important and then choose a method to solve the problem, which could vary from student to student. As a result, this assessment component involved more critical thinking, critical writing, and critical reading—along with more opportunities to build mathematics self-efficacy—as compared to the original section of the course.

Data Analysis

We examined three different indicators of student performance to assess the full impact of the pilot Intermediate Algebra class: (a) the extent to which students persisted in the course by comparing percentage retention and dropouts (i.e., non-completers) by section; (b) student performance in the course by comparing specific and total grade components by section; and (c) long-term effects of the intervention by comparing student performance in their next semester's College Algebra course by prerequisite section (original vs. pilot). Because this last piece was most trenchant to this study's primary hypothesis, student performance in College Algebra by ethnicity between sections was also examined.

Further, we analyzed seven potential predictors of College Algebra grade—Intermediate Algebra section, Intermediate Algebra grade, Math ACT score, CCHE index, student gender, student status (e.g., freshman, transfer), and student ethnicity—via multiple regression, the most powerful way to minimize the problem of multicolinearity (i.e., significant intercorrelations among the variables). An exploratory approach using forward selection followed by backward elimination was implemented, yielding a final regression model for the predictors of College Algebra grades.

Finally, student satisfaction with the Intermediate Algebra course was investigated using a standardized teaching and course evaluation, the Teacher Behaviors Checklist (TBC; Keeley, Smith, & Buskist, 2006), which has demonstrated excellent reliability (e.g., a Cronbach's alpha of .95) and construct validity to discriminate between good versus bad teachers from students' perspectives (Keeley, Furr, & Buskist, 2010). The TBC was administered online and asked students to rate the instructor on a 5-point Likert scale—ranging from "almost never" to "almost always"—regarding the extent to which she or he possessed each of 11 different traits/qualities. The instructor qualities included being approachable, professional, creative and interesting, and technologically competent. Each quality was followed by a list of sample corresponding behaviors in parentheses; for example,

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"Approachable/Personable (smiles, greets students, initiates conversations, invites questions, responds respectfully to student comments)."

Results

Student Performance Indicators

There were no significant differences in student performance between the pilot and original sections of the Intermediate Algebra course (see Table 4, p. 32). The rates of low grades (D, F, or W) were virtually identical (40% in each section), as were the "disappearance" rates—that is, students who did not complete the course due to absences or withdrawals (29% pilot vs. 31% original course). There were no significant section differences in any of the grade components—homework, writing, or portfolios—and the final Intermediate Algebra course grade average was 75% in both sections. Further, the students' cumulative GPAs in all their courses combined during the Fall 2009 semester were indistinguishable. There was, however, a nonsignificant trend for the pilot students to perform better on their class tests in Intermediate Algebra than the original students (see Table 4, p. 32).

Table 3

Content Order Differences in the Original vs. Pilot Intermediate Algebra Course

Original course order	Pilot course order
One-variable linear equations • solving problems such as: $3(x–4)+12=19$ • solving inequality problems and graphing on the number line such as: $3x–4<12$ *Two-variable linear functions* • understanding linear functions • finding slope and intercepts • graphing linear inequalities in two variables *Systems of equations* • understanding linear systems wherein there is no solution, one solution or infinite solutions *Exponents* • simplifying expressions with laws of exponents (power rule, quotient rule, product rule) • radical and rational exponent notation *Polynomial expressions* • adding, subtracting, multiplying expressions (e.g., $3x–2$ and $4x+8$) • removing the greatest common factor and factoring of simple trinomials *Quadratic functions* • finding intercepts using the quadratic formula and factoring	*Exponents* • simplifying expressions with laws of exponents (e.g., power rule, quotient rule, product rule) • radical and rational exponent notation *Polynomial expressions* • adding, subtracting, multiplying expressions (e.g., $3x–2$ and $4x+8$) • removing the greatest common factor and factoring of simple trinomials • reinforcing how polynomial expressions use the laws of exponents *One-variable linear equations* • solving problems such as: $3(x–4)+12=19$ • solving inequality problems and graphing on the number line such as: $3x–4<12$ *Solving equations with a functions approach* • emphasis on one solution vs. multiple solutions in context (such as $x>94$ means a test score of 95, 96, 97, etc) • finding horizontal intercepts of each function • analyzing different rates of change in context • emphasis on real world problems *Systems of equations* • understanding linear systems wherein there is no solution, one solution or infinite solutions • understanding linear and quadratic systems with two solutions and eliminate unreasonable solution (e.g., negative number of ducks)

Table 5 (p. 32) shows the results of our main hypothesis: that the pilot course would better prepare students for their future college-level mathematics classes. The most common next mathematics class following Intermediate Algebra is College Algebra, which was taken by 45% of the original students and 53.5% of the pilot students in Winter 2010. The pilot students performed significantly better in College Algebra, earning almost a full letter grade higher than the original students (GPA of 2.70 vs. 1.78, $t(37) = 2.18$). This difference was especially pronounced for Native-American students, who earned more than two letter grades higher on average if they had taken the pilot section of Intermediate Algebra instead of the original section as a prerequisite to College Algebra the prior semester (GPA of 3.60 vs. 1.33, $t(6) = 3.87$).

The final regression model for the grade predictor analysis showed three predictor variables—Intermediate Algebra course section, Intermediate Algebra grade, and CCHE index—accounted for 53% of the variance in student grades in their subsequent College Algebra course the following semester (see Table 6, p. 33). The other four potential predictor variables—Math ACT score, student gender, student status (e.g., freshman, transfer), and student ethnicity—did not significantly predict grades in College Algebra (all $ps > .17$).

In light of the significant findings regarding student success in College Algebra, we wanted to investigate whether the pilot course had any generalizable and continuing effects for Native-American students—the target of the Title III grant—when it was subsequently taught by several different FMP instructors (in addition to the first author). To accomplish this, we examined data on College Algebra grades for Native-American students in Winter 2009 (in which none of the students had taken the pilot Intermediate Algebra course as a prerequisite) and Winter 2011 (in which most of the students had taken the pilot Intermediate Algebra course as a prerequisite). We then conducted an ANOVA to determine whether Native-American student grades in College Algebra differed by prerequisite course type—that is, original Intermediate Algebra, pilot Intermediate Algebra, or none (student placed into College Algebra). The overall ANOVA revealed a trend toward significance, with $F(2, 113) = 2.58, p = .08$, such that Native-American students who took the pilot Intermediate Algebra (TRS 92) course outperformed those who took the original Intermediate Algebra (TRS 92) course as a prerequisite to College Algebra (see Table 7, p. 34). In fact, the students who took the pilot Intermediate Algebra course performed almost as well as those Native-American students who placed into College Algebra via their test scores and had not taken any developmental mathematics beforehand (see Figure 1, page 35).

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Student Satisfaction

There were no significant differences in student ratings of the instructor by section on any of the evaluation items from the Teacher Behaviors Checklist (Keeley, Smith, & Buskist, 2006). Yet there was a nonsignificant trend for students in the pilot section to rate the instructor lower across the board, particularly on "creative and interesting" and "passionate." The average rating out of 5 for pilot students was about 3.5 for each of these items versus 4.0 for original students (*ps* = .10 and .17 respectively), despite the fact that the same instructor (LG) taught each course section. There were no significant between-section differences in student ratings of their expected course grade nor of how applicable the students believed various course components (study skills, writing, and mathematics) were to their future college career. However, there was a large difference in reported textbook usage, such that pilot students endorsed using the course textbook significantly more than students in the original course (means of 3.08 vs. 2.25, $t(63) = 2.62$, $p = .01$, $d = .75$).

Table 4

Intermediate Algebra Percentage Grades by Section (Original vs. Pilot) in Fall 2009

Grade component	Section	N	M %	SD	p
Homework	Original	20	80.91	17.97	.29
	Pilot	56	77.11	11.90	
Writing/journal	Original	20	74.29	16.88	.92
	Pilot	56	74.90	23.84	
Tests	Original	20	72.03	14.70	.15
	Pilot	56	76.62	11.13	
Portfolio	Original	20	73.15	22.41	.42
	Pilot	56	67.08	30.92	
FINAL GRADE	Original	20	74.93	11.28	.96
	Pilot	56	75.08	12.11	
TERM GPA[a]	Original	20	2.83	0.90	.94
	Pilot	56	2.82	0.73	

[a] Term GPA is the student's cumulative GPA in all their courses combined during the Fall 2009 semester.

Table 5

College Algebra Grades in Winter 2010 by Prerequisite Section (Original vs. Pilot)

Section	N	M GPA[a]	SD	p	d
Original	9	1.78	1.48	.04	0.83
Pilot	30	2.70	0.99		
Original (Native Americans only)	3	1.33	1.15	.01	2.83
Pilot (Native Americans only)	5	3.60	0.55		

[a] Mean GPA is only for the College Algebra course, with *A*=4; *B*=3; *C*=2; *D*=1; and *F* or *W*=0; *d*=Cohen's (1992) effect size, where any effect above 0.80 is considered "large."

Discussion

The success of students' in the redesigned pilot Intermediate Algebra sections and greater pass rates in the next college-level mathematics course reflects positive support for all research questions. The redesign work was intended to improve the rigor of the Intermediate Algebra course, and thereby increase the cognitive demand for students, to teach skills and concepts in a more cohesive manner, and to build the students' interest and self-efficacy in mathematics. The changes to the structure of the course, the content of the course, and the assessments were based on current philosophies in developmental mathematics education (Armington, 2003; Goolsby, 1988; Weens, 1998). Applying these philosophies showed positive benefits for pilot students.

Study participant demographics indicate no notable pre-existing differences between students by course section (pilot or original) in gender, ethnicity, standardized test scores, or success predictors, although there was a significant difference in the number of freshman (25% more in the pilot class) due to the nonrandom assignment of students to groups (i.e., they were able to register online for whichever class best fit their schedule). Therefore, the lack of differences in dropouts, overall grades, or on any grade component in the students' Intermediate Algebra performance cannot be attributed to prior differences between subjects. This is especially salient in light of the *Changing Equations* large-scale redesign project, which showed that scores on direct measures of student learning (common exams, pre/posttests) went up for students in the pilot sections but completion rates (final grades of C or better) went down (Twigg, 2011). Completion rates in the resigned course described herein did not change.

There was a nonsignificant trend toward lower student satisfaction with the instructor in the pilot sections; this may be due to the fact that the students in the pilot sections were assigned more frequent and more challenging mathematics work than their peers in the original course section. In accordance, the pilot students reported significantly more textbook usage throughout the semester, approaching a large effect size ($d = .75$). Increased text use could indicate that pilot students were demonstrating more self-directed learning of basic concepts, which may have enabled these students to develop higher mathematics self-efficacy

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as well as freed up more class time to be spent on higher-order thinking.

The most noteworthy result was the significant difference in student performance in College Algebra the following semester, with large effect sizes ranging from $d = .83$ to $d = 2.83$ ($d = .80$ or above is large in Cohen's 1992 classification). This was especially pronounced for Native-American students, thus showing a powerful "sleeper" or delayed effect of the new Intermediate Algebra pilot course. This same pattern persisted into more recent semesters in which several different FMP faculty members began to teach the Intermediate Algebra pilot course. Specifically, Native-American students who had taken the pilot Intermediate Algebra course with any FMP instructor earned higher grades in College Algebra than those who had taken the original Intermediate Algebra course and performed almost as well as those who had placed into College Algebra via their test scores without having taken any developmental mathematics prerequisite course. The salutary effect of this redesigned course on Native-American students may have occurred because of the course's explicit focus on individualized learning (e.g., via real-time instructor feedback on Math XL work), group work, and more integrated content, with a capstone problem and optimally-ordered topics. These changes may have increased students' self-efficacy, which has been shown to be a critical factor in raising Native American levels of mathematics performance (House, 2009).

Table 6

Final Regression Model for Predictors of Student Grade in College Algebra

Variable	*B*	*SE B*	β
Section (Original vs. Pilot)	−1.11	.42	−.37*
Intermediate Algebra grade[a]	0.46	.19	.34*
CCHE Index[b]	−0.01	.02	−.36*

Note. $N = 28$; $R^2 = .53$.
[a] Grade coded as ordinal, with A=4; B=3; C=2; D=1; F or W=0.
[b] CCHE Index is a composite score assigned by the Colorado Commission on Higher Education to indicate college readiness.
* $p < .05$.

Limitations

One limitation of the current study was its quasi-experimental nature, with nonrandom assignment of students to class sections. There have been instances at our college wherein different class sections were unequal in terms of student motivation or achievement levels, which could have biased the research results. However, the fact that there were no key pre-existing differences between sections in terms of achievement (ACT, CCHE index scores), gender, or ethnicity bolsters confidence in the validity of the present findings.

Another limitation to the study's generalizability is that only one instructor (LG) taught all three pilot sections. This was done in order to minimize instructor variance, although it is feasible that the course's long-term effect occurred because the pilot content is an optimal fit for the particular instructor who designed it. However, analysis of the additional data from Winters 2009 and 2011 suggests that the pilot course's effects generalize beyond the specific course instructor, at least for Native-American students.

Implications for Future Research and Practice

The strongest, and somewhat surprising, outcome of the present study was the improved success of Native-American students in their subsequent College Algebra course. This finding points to the need for a larger replication study to shed light on whether and why Native Americans differentially benefit from a redesigned collaborative-learning curriculum in developmental mathematics. If the sustained effect found in this study can be replicated, then it would be important to identify the critical ingredients underlying that effect. Future research should therefore examine potential mediators such as student levels of self-efficacy, problem-solving, critical thinking, or other intermediary variables in addition to ethnicity. It would also be useful to investigate the effect of the varied strategies implemented in this pilot, especially those that are most resource-intensive, to determine if all are necessary to produce the positive results.

In this same vein, future studies could evaluate the role that campus support services play in the success of developmental mathematics courses and programs (e.g., Fowler & Boylan, 2010). For instance, our college has a Native-American student center and a new STEM-cubed program to serve minority and first-generation college students. Both of these incorporate all three strategies recommended by Guillory (2009) to improve Native-American student retention, offering specialized academic advising, peer mentoring, and free tutoring and Supplemental Instruction in mathematics (and other) courses across the curriculum. In addition, campus support services may benefit by applying some of the specific ingredients described in this redesign, such as instructor feedback on Math XL work and a scaffolding approach, in their tutoring centers. Empirical data to support the value of such programs could be instrumental to their ongoing funding and vitality. Moreover, it remains to be determined whether these effects can be replicated with other underserved and minority populations such as Hispanics, African Americans, and students of low socioeconomic status.

Finally, future research should investigate whether the success of this pilot Intermediate Algebra course will exert a noticeable effect on college retention and graduation. It could be valuable to examine whether this and other redesigned developmental mathematics courses make a dent in the funereal image of developmental mathematics as a major obstacle to college success.

Overall, several critical implications of this research for the practice of developmental mathematics—and developmental education in general—are clear: First, undertaking the often arduous and time-consuming task of course redesign may pay significant dividends down the road, especially for underserved populations demonstrating less than optimal performance under the present curriculum. Second, course rigor can be increased without compromising student success even with high-risk populations. Tailoring course content, structure, and assessment to what research reveals about how students learn can help them master more challenging content, thereby bolstering their academic success in future college-level courses. Such course redesigns may necessitate a philosophical rethinking of developmental mathematics teaching strategies (e.g., Armington, 2003) to emphasize more problem-based, collaborative, and student-centered forms of teaching and learning rather than traditional classroom lecture. Further, if the Native American performance boost reported herein turns out to be robust and reliable, then institutions that serve Native Americans would do well to review the design of their developmental mathematics classes and to what extent they could be improved or retooled to optimize student success.

Course rigor can be increased without compromising student success even with high-risk populations.

Two other potential implications for practice emerge from findings regarding differential textbook usage and instructor/course evaluations between sections. Students in the pilot section of Intermediate Algebra reported using their textbooks significantly more frequently throughout the semester, which may have contributed to their better preparation for College Algebra the next semester. Although a number of studies have examined the use of mathematics textbooks by teachers (e.g., Johansson, 2006; Remillard, 2005), there is a dearth of research into the use of mathematics textbooks by students (Love & Pimm, 1996; Rezat, 2009). However, one element that has been shown to enhance student textbook usage in their mathematics course has been problem-based and self-directed learning, which require students to search for answers on their own first before getting assistance from the instructor (Rezat, 2009). This closely resembles two aspects of the pilot course described in this study: the scaffolding approach used in class and the revised problem-solving instructions requiring students to document their attempted solutions to a given problem in writing. Either or both of these strategies may have contributed to increased textbook usage by the students, which in itself would be a great benefit to any instructor.

Second, there was a nonsignificant trend toward pilot students reporting less satisfaction with the course than original students. If this pattern persists, then it could be troubling for the developmental instructor, especially given the importance placed on teaching evaluations by many administrators in hiring and tenure decisions. A glance at the subsequent student evaluations of the first author (LG), however, suggests that the trend did not continue: The overall instructor rating for LG (out of 5) was 3.67 ($N = 49$) for the Fall 2009 pilot course sections, 3.93 ($N = 14$) for the Fall 2009 original course section, and 4.22 ($N = 67$) for her 2010-2011 pilot course sections combined, with the most recent pilot course offering (Summer 2011) producing a DFW rate of a mere 11%. Thus, the lower instructor ratings for the Fall 2009 pilot course may have been due to one-time factors rather than any persistent student dissatisfaction per se. One possibility is that it was an artifact of the instructor's adjustment to teaching new material for the first time. Another possibility is that Fall 2009 pilot students were comparing themselves to their peers in the original course that semester who were generally doing less work; this

Table 7

Native-American College Algebra Grades by Prerequisite Class (Winters 2009, 2011)

Section	*N*	*M* GPA[a]	*SD*
Original	35	1.43	1.22
Pilot	19	1.95	1.31
Placed into College Algebra[b]	62	2.02	1.25

[a] Mean GPA is only for the College Algebra course, with A=4; B=3; C=2; D=1; and F or W=0.
[b] Student tested into College Algebra via ACT score of 19+ or Accuplacer equivalent.

comparison is no longer occurring because only the pilot (redesigned) course is currently being offered.

Conclusion

Experimenting with more effective course design strategies for developmental mathematics classes has a potential to have a sustained impact on college persistence. The present research suggests that, without changing course content, a developmental class can be altered to be more effective at its primary goal—preparing students to learn future college-level material. The primary changes in the redesigned Intermediate Algebra course described herein were three-fold and involved: (a) a reordering of content and addition of a capstone problem in order to make the material more meaningful and applied; (b) increasing the cognitive demand of the course by requiring students to work harder in and out of class (e.g., scaffolding), which was accomplished without a concomitant increase in the failure rate; and (c) a specific emphasis on building students' mathematical self-efficacy by providing more self-directed and collaborative learning opportunities with frequent instructor feedback. Data analysis suggests that these changes benefitted students overall—and perhaps especially Native-American students—both in Intermediate Algebra and next college-level mathematics course. This pilot research study should be expanded as it addresses key goals shared by individual students, postsecondary professionals, and the nation: helping underserved students succeed in college-level courses.

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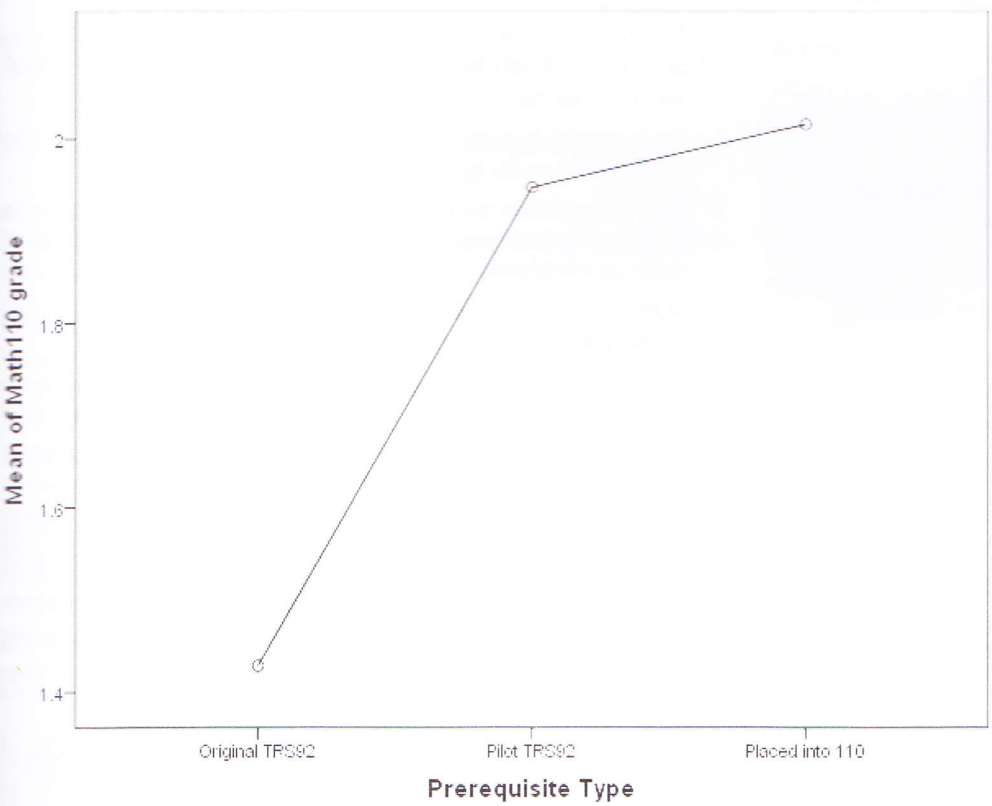

Figure 1. **Native-American College Algebra grades by prerequisite class (Winters 2009, 2011).**

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Critical Thinking Principle

To settle a question, one must know what it is asking and how to go about answering it. In other words, for every question one might ask, are there conditions that must be met before the question can be settled?

Performance Indicators and Dispositions

Students who think critically seek a clear understanding of the main question they are trying to answer, problem they are trying to solve, or issue they are trying to resolve. They formulate questions clearly and precisely. They recognize when they are dealing with a complex question, and they think deeply within its complexities before attempting to answer such a question.

Outcomes

1. Students express in their own words (clearly and precisely) the question at issue (in a lesson, chapter, assignment, etc.).
2. Students re-express a question in a variety of ways (with clarity and precision).
3. Students divide complex questions into subquestions (accurately delineating the complexities in the issue).
4. Students formulate foundational and significant questions within any particular discipline or subject.
5. Students accurately categorize the question before reasoning through it, determining whether it is a question of fact or preference, or one that calls for reasoned judgment.
6. Students distinguish conceptual questions from factual questions.
7. Students distinguish significant questions from trivial ones, relevant from irrelevant ones.
8. Students demonstrate sensitivity to the assumptions built into the questions they ask; they analyze and assess those assumptions for justifiability.
9. Students distinguish questions they can answer from those they cannot answer.

Standard Three: Information, Data, Evidence, and Experience

Students who think critically recognize that all thinking is based on some data, information, evidence, experience, or research.

Critical Thinking Principle

Thinking can only be as sound as the information upon which it is based.

Performance Indicators and Dispositions

Students who think critically seek the information relevant to the questions they are trying to answer, problems they are trying to solve, or issues they are trying to resolve. They routinely check information for accuracy. They make sure they are considering all of the important information before attempting to answer a question and that the information they have is sufficient to answer the question. Students who think critically also routinely analyze and assess the information used by others (using the same guidelines).

Outcomes

1. Students express in their own words (clearly and precisely) the most important information (in a discussion, chapter, assignment, etc.).
2. Students distinguish the following related but different concepts: facts, information, experience, research, data, and evidence.
3. Students state their evidence for a view clearly and fairly.
4. Students distinguish relevant from irrelevant information when reasoning through a problem. They consider only relevant information, disregarding what is irrelevant.
5. Students actively search for information against, not just for, their own position.
6. Students draw conclusions only to the extent that those conclusions are supported by the facts and sound reasoning. They demonstrate the ability to objectively analyze and assess information in coming to conclusions based on the information.
7. Students demonstrate understanding of the difference between information and inferences drawn from that information. They routinely delineate information and inferences in their own and others' reasoning.
8. Students demonstrate understanding of the types of information used within particular subjects and disciplines, as well as understanding of how professionals within fields use information in reasoning through problems.

Conclusion

In this column we have focused on three general critical thinking competency standards. These competencies are relevant to thinking well within any subject, discipline, or domain of thought. In the next few columns we will focus on additional general competencies in critical thinking, along with several subject-specific critical thinking competencies.

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