**Day 9: Linear Systems – Pre-Notes**

There are two algebraic methods to solve a linear system. They are substitution and elimination.

***Substitution Method***

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| --- | --- |
| STEP 1 | Choose an equation and solve for one variable in terms of the other variable. |
| STEP 2 | Substitute the expression from **STEP 1** into the other equation. |
| STEP 3 | Solve for the unknown variable. |
| STEP 4 | To solve for the other variable, substitute the solution from **STEP 3** into the equation found in **STEP 1**. |
| STEP 5 | Write an ordered pair of the two found values |
| STEP 6 | Check the solution in both original equations. |

**Example 1**: Solve the system of equations by using substitution.

$$\left\{\begin{array}{c}2x+y=1\\3x+4y=14\end{array}\right.$$

STEP 1:

After choosing the first equation: $2x+y=1$, solve for y

$$y=1-2x$$

STEP 2:

Substitute the expression from STEP 1 into the other equation.

$$3x+4y=14$$

$$3x+4(1-2x)=14$$

STEP 3:

Solve for the unknown variable (*x* in this case).

$$3x+4(1-2x)=14$$

$3x+4-8x=14$ *Distribute the 4.*

$4-5x=14$ *Combine like terms.*

$-5x=10$ *Subtract 4 from both sides*.

$x=-2$ *Divide both sides by −5.*

STEP 4:

To solve for the other variable, substitute the solution from **STEP 4** into the equation found in **STEP 1**.
$$y=1-2x$$

$$y=1-2(-2)$$

$$y=1+4$$

$$y=5$$

STEP 5:

Write an ordered pair of the two found values.

(−2, 5)

STEP 6:

Check the solution in both original equations.

|  |  |
| --- | --- |
| First Equation: $2x+y=1$ | Second Equation: $3x+4y=14$ |
| $$2(-2)+5=1$$$$-4+5=1$$$$1=1$$ | $$3(-2)+4(5)=14$$$$-6+20=14$$$$14=14$$ |

**Example 2**: Solve the system of equations by using substitution.

$$\left\{\begin{array}{c}y=x+4\\y=-2x+7\end{array}\right.$$

STEP 1:

Not necessary b/c both equations are written as *y*=. Choose the first equation: $y=x+4$

STEP 2:

Substitute the expression from STEP 1 into the other equation.

$$y=-2x+7$$

$$x+4=-2x+7$$

STEP 3:

Solve for the unknown variable (*x* in this case).

$$x+4=-2x+7$$

$3x+4=7$ *Add 2x to both sides.*

$3x=3$ *Subtract 4 from both sides*

$x=1$ *Divide both sides by 3*

STEP 4:

To solve for the other variable, substitute the solution from **STEP 4** into the equation found in **STEP 1**.
$$y=x+4$$

$$y=1+4$$

$$y=5$$

STEP 5:

Write an ordered pair of the two found values.

(1, 5)

STEP 6:

Check the solution in both original equations.

|  |  |
| --- | --- |
| First Equation: $y=x+4$ | Second Equation: $y=-2x+7$ |
| $$5=1+4$$$$5=5$$ | $$5=-2(1)+7$$$$5=-2+7$$$$5=5$$ |

**Question 1:**

Solve the system of equations by using substitution.

$$\left\{\begin{array}{c}y=x+3\\5y-2x=21\end{array}\right.$$

Solution: (2, 5)

**FAQ: When do I use the substitution method?**

**Answer: When one variable is already solved for OR it would be very easy to solve for a variable.**

***Elimination Method***

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| --- | --- |
| STEP 1 | Choose a variable in the equations to eliminate. |
| STEP 2 | If necessary, multiply one or both equations by a number that will make the coefficients of one of the variables in the equations the same but with opposite signs. |
| STEP 3 | Add the equations together to eliminate one of the variables. |
| STEP 4 | Solve for the unknown variable. |
| STEP 5 | To solve for the other variable, substitute the solution from **STEP 4** into either equation and solve for the other variable. |
| STEP 6 | Write an ordered pair of the two found values. |
| STEP 7 | Check the solution in both original equations. |

**Example 3**: Solve the system of equations by using elimination.

$$\left\{\begin{array}{c}3x+2y=14\\5x-2y=18\end{array}\right.$$

STEP 1 and 2:

Choose to eliminate the variable *y* since the coefficients are the same with opposite signs. No need to multiply either equation.

STEP 3:

Add the equations together to eliminate the variable *y*. Notice that $2y+\left(-2y\right)=0$.

$$\genfrac{}{}{0pt}{}{ 3x+2y=14}{+5x-2y=18}$$

$$ 8x=32$$

STEP 4:

Solve for the unknown variable.

$$8x=32$$

$x=4$ *Divide both sides by 8*

STEP 5:

To solve for the other variable, substitute the solution from **STEP 4** into either equation and solve for the other variable.

$$3x+2y=14$$

$$3(4)+2y=14$$

$$12+2y=14$$

$$2y=2$$

$$y=1$$

STEP 6:

Write an ordered pair of the two found values.

(4, 1)

STEP 7:

Check the solution in both original equations.

|  |  |
| --- | --- |
| First Equation: $3x+2y=14$ | Second Equation: $5x-2y=18$ |
| $$3(4)+2(1)=14$$$$12+2=14$$$$14=14$$ | $$5(4)-2(1)=18$$$$20-2=18$$$$18=18$$ |

**Example 4**: Solve the system of equations by using elimination.

$$\left\{\begin{array}{c}2x+y=1\\3x+4y=14\end{array}\right.$$

STEP 1 and 2:

Choose to eliminate the variable *y* because then you only need to multiply the first equation. Multiply the ENTIRE first equation by −4 so that the coefficients of *y* in the equations are the same but with opposite signs.

$\begin{array}{c}-4(2x+y=1)\\3x+4y=14\end{array}$ **🡪** $\begin{array}{c}-8x-4y=-4\\3x+4y=14\end{array}$

STEP 3:

Add the equations together to eliminate the variable *y*. Notice that $-4y+4y=0$.

$$\begin{array}{c}-8x-4y=-4\\ 3x+4y=14\end{array}$$

$$-5x=10$$

STEP 4:

Solve for the unknown variable.

$$-5x=10$$

$$x=-2$$

STEP 5:

To solve for the other variable, substitute the solution from **STEP 4** into either equation and solve for the other variable.

$$2x+y=1$$

$$2(-2)+y=1$$

$$-4+y=1$$

$$y=5$$

STEP 6:

Write an ordered pair of the two found values.

(−2, 5)

STEP 7:

Check the solution in both original equations.

|  |  |
| --- | --- |
| First Equation: $2x+y=1$ | Second Equation: $3x+4y=14$ |
| $$2(-2)+5=1$$$$-4+5=1$$$$1=1$$ | $$3(-2)+4(5)=14$$$$-6+20=14$$$$14=14$$ |

**Question 2:**

Solve the system of equations by using elimination.

$$\left\{\begin{array}{c}3x+4y=10\\4x-2y=6\end{array}\right.$$

Solution: (2, 1)

**FAQ: When do I use the elimination method?**

**Answer: When it would not be easy to solve for a variable. Many times this means that both the *x* and *y* terms have coefficients that are not equal to 1 or -1.**