Cosmic Neutrinos and New Physics beyond the Electroweak Scale

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New physics beyond the electroweak scale may increase weak interaction cross sections beyond the Standard Model predictions. Such cross sections can be expected within theories that solve the hierarchy problem of known interactions with a unification scale in the TeV range. We derive constraints on these cross sections from the flux of neutrinos expected from cosmic ray interactions with the microwave background and the non-observation of horizontal air showers. We also discuss how this limit can be improved by upcoming cosmic ray and neutrino experiments, and how the energy dependence of the new interactions can be probed by these experiments.

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I. INTRODUCTION

It has been suggested that the neutrino-nucleon cross section, $\sigma_{\nu N}$, can be enhanced by new physics beyond the electroweak scale in the center of mass (CM) frame, or above about a PeV in the nucleon rest frame. At ultra-high energies (UHE), neutrinos can in principle acquire νN cross sections approaching hadronic levels. It may therefore be possible for UHE neutrinos to initiate air showers, which would offer a very direct means of probing these new UHE interactions. The results of this kind of experiment have important implications for both neutrino astronomy and high energy physics.

For the lowest partial wave contribution to the cross section of a point-like particle, this new physics would violate unitarity [1]. However, two major possibilities have been discussed in the literature for which unitarity bounds need not be violated. In the first, a broken SU(3) gauge symmetry dual to the unbroken SU(3) color gauge group of strong interaction is introduced as the "generation symmetry" such that the three generations of leptons and quarks represent the quantum numbers of this generation symmetry. In this scheme, neutrinos can have close to strong interaction cross sections with quarks. In addition, neutrinos can interact coherently with all partons in the nucleon, resulting in an effective cross section comparable to the geometrical nucleon cross section. This model lends itself to experimental verification through shower development altitude statistics, as described by its authors [2]. The present paper will not affect the plausibility of this possibility, which largely awaits more UHE cosmic ray (UHECR) events to compare against.

The second possibility consists of a large increase in the number of degrees of freedom above the electroweak scale [3]. A specific implementation of this idea is given in theories with n additional large compact dimensions and a quantum gravity scale $M_{4+n} \sim$ TeV that has recently received much attention in the literature [4] because it provides an alternative solution (i.e., without supersymmetry) to the hierarchy problem in grand unifications of gauge interactions. For parameters consistent with measured cross sections at electroweak energies and below, at $\simeq 10^{20}$ eV, the νN cross section can approach $10^{-4} - 10^{-2}$ times a typical strong cross section [5]. Therefore, in these models the mean free path for νN interactions at these energies is on the order of kilometers to hundreds of kilometers in air. Experimentally, this would be reflected by a specific energy dependence of the typical column depth at which air showers initiate. As we will show, this leads to a useful signature, deriving from zenith angle variations with shower energy. Comparison with observations would either support these scenarios or constrain them in a way complementary to studies in terrestrial accelerators [6] or other laboratory experiments that have recently appeared in the literature.

Both scenarios for increasing νN cross sections raise the question of how far UHE neutrinos can travel in space. In the first case, due to the *coherent* nuclear interactions, the cross section approaches the nucleon-nucleon cross section, and so intergalactic matter becomes the greatest threat to the neutrino traveling long distances unimpeded. However, an order of magnitude calculation shows that the mean free path for such a neutrino is $\sim 10^2$ Gpc, if we simply assume the matter density of the Universe is its critical density. Within a galaxy with similar interstellar density to the Milky Way, the mean free path is $\sim$ Mpc; the UHE neutrino is not attenuated before reaching Earth's atmosphere. For the extra dimension model, the cross sections are lower, so interaction with matter is not a problem. With Cosmic Microwave Background (CMB) photons, the CM energy is too low to give rise to a significant interaction probability. If neutrinos are massive, then the relic neutrino background is more of a threat. But assuming the neutrino mass is below 92 eV, as conservatively required to prevent over-closing the Universe for a stable species [7], the mean free path is $\sim 10^4$ Gpc at a CM energy of $\sim 100\,\mathrm{GeV}$, corresponding to a $\sim 10^{20}\,\mathrm{eV}$ neutrino. The cosmological horizon is only ~ 15 Gpc, so for both scenarios the neutrinos can reach Earth unhindered from arbitrarily great distances.

A major motivation for attempts to augment the νN cross section at ultra-high energies derives from the paradox of UHECRs. Above $\sim 10^{20}$ eV, protons and neutrons see CMB photons doppler shifted to high energy gamma rays, and encounters generate pions, thereby reducing the resulting nucleon's energy by at least the pion rest mass. One therefore expects the Greisen-Zatespin-Kuzmin (GZK) cutoff [8], wherein cosmic rays above $E_{\mathrm{GZK}} \simeq 7 \times 10^{19}$ eV would lose their energy within $\simeq 50\,\mathrm{Mpc}$. However, since we observe cosmic rays at super-GZK energies, it has been proposed that these UHECRs might really be neutrinos rather than nucleons (for example, see [2]). While this is a strong motivation to explore new UHE neutrino interactions, we will not need to assume that UHECRs are neutrinos.

The paper is outlined as follows: In Section II we derive a constraint on the neutrino-nucleon cross section resulting from the non-observation of horizontal air showers induced by neutrinos produced by interactions of the highest energy cosmic rays with the cosmic microwave background. In Section III we discuss potential improvements of this constraint including other hypothesized neutrino sources and next generation experiments. In Section IV we narrow the discussion to the case of a νN cross section scaling linearly with CM energy, as an example. A zenith angle profile is presented as a signature of this scaling, and a means of constraining it. We conclude in Section V.

II. A BOUND FROM THE "COSMOGENIC" NEUTRINO FLUX

The easiest way to distinguish extensive air showers (EAS) caused by neutrinos from those caused by hadrons is to look for nearly horizontal electromagnetic showers. As pointed out in Ref. [9], the Earth atmosphere is about 36 times thicker taken horizontally than vertically. A horizontally incident UHE hadron or nucleus has a vanishingly small chance of descending to sufficiently low altitudes to induce air showers detectable by ground instruments. As a consequence, a horizontal shower is a very strong indication of a neutrino primary.

The two major techniques for observing extensive air showers produced by UHECRs or neutrinos are the detection of shower particles by an array of ground detectors, and the detection of the nitrogen fluorescence light produced by the shower. The largest operating experiment utilizing the first technique is the AGASA which covers about $100\,\mathrm{km}^2$ in area [10]. The second technique was pioneered by the Fly's Eye instrument [11] and produced a total exposure similar to the AGASA instrument. For cosmic neutrino detection, the fluorescence technique may provide somewhat more detailed information, because it can readily measure the position of the shower maximum and possibly the first interaction, which are important in distinguishing neutrino primaries. Also, fluorescence detectors are equipped to see horizontal showers that do not intersect the ground near the detector.

The main backgrounds consist of muon bremsstrahlung and tau lepton decays, as these could occur very deep in the atmosphere. The flux of these particles is several orders of magnitude below the CRs and predicted neutrinos of the same energy, but if UHE neutrinos interact with matter only through Standard Model channels, then the rate of neutrino-induced triggers will still be dwarfed by these background events. The backgrounds for high zenith angle events differ somewhat between fluorescence detectors and ground arrays [12,13], but both detector classes have remedies for the high background. For example, one interesting benefit with air fluorescence is the possibility of seeing a separate primary CR shower associated with the event, thus identifying it as a muon- or tau-induced secondary [14]. Ground arrays would not see this sign, but they should be able to discriminate between neutrinos and secondaries by the shape of the shower front [15].

The strategy here is to use observations (or non-observation) of horizontal showers to place limits on the UHE neutrino flux incident on the Earth. Comparing that against a reliably predicted flux permits us to bound the cross section.

Fig. 1 summarizes the high energy neutrino fluxes expected from various sources, where the shown flavor summed fluxes consist mostly of muon and electron neutrinos in a ratio of about 2:1. Shown are the atmospheric neutrino flux [16], a typical flux range expected for "cosmogenic" neutrinos created as secondaries from the decay of charged pions produced by collisions between UHE nucleons and CMB photons [17], a typical prediction for the diffuse flux from photon optically thick proton blazars [18] that were normalized to recent estimates of the blazar contribution to the diffuse $\gamma-$ray background [19], and a typical prediction by a scenario where UHECRs are produced by decay of particles close to the Grand Unification Scale [20] (for a review see also Ref. [21]).

Apart from the atmospheric neutrino flux, only the cosmogenic neutrinos are nearly guaranteed to exist due to the known existence of UHECRs, requiring only that these contain nucleons and be primarily extragalactic in origin. This is an assumption, since Galactic sources can not be wholly ruled out [22]. However, if the charge of the primary UHECR satisfies $Z \lesssim \mathcal{O}(1)$ (which will be tested by forthcoming experiments), an extragalactic origin is favored on the purely empirical grounds of the observed nearly isotropic angular distribution which rules out Galactic disk sources.

The reaction generating these cosmogenic neutrinos is well known, depending only on the existence of UHE nucleon sources more distant than $\lambda_{\rm GZK} \simeq 8$ Mpc and on the relic microwave photons known to permeate the Universe. The two cosmogenic lines in Fig. 1 were computed for a scenario where radiogalaxies explain UHECRs between $\simeq 3\times 10^{18}$ eV (i.e. above the "ankle") and $\simeq 10^{20}$ eV [31], with source spectra cutoffs at 3×10^{20} eV (lower curve) and 3×10^{21} eV (upper curve) [17]. Since events were observed above 3×10^{20} eV, this may be used as a conservative flux estimate (see also Sect. III below).

The non-observation of horizontal showers by the neutrino fluxes indicated in Fig. 1 can now be translated into an upper limit on the total neutrino-nucleon cross section. The total charged current Standard Model νN cross section can be approximately represented by [32]

$$\sigma_{\nu N}^{SM}(E) \simeq 2.36\times 10^{-32}(E/10^{19}\,{\rm eV})^{0.363}\,{\rm cm}^2 \quad (10^{16}\,{\rm eV} \lesssim E \lesssim 10^{21}\,{\rm eV})\,. \tag{1}$$

In any narrow energy range and as long as the neutrino mean free path is larger than the linear detector size, the neutrino detection rate scales as [9]

$$\text{Event Rate} \propto A\,\phi_\nu\,\sigma_{\nu N}\,, \tag{2}$$

where A is the detector acceptance (in units of volume times solid angle) in that energy band, and ϕ_ν is the neutrino flux incident on the Earth. In our case, the non-observation of neutrino-induced (horizontal) air showers limits the event rate, and thereby sets the experimental upper bounds given in Fig. 1. On the other hand, the gap between the best current experimental flux bound and the predicted cosmogenic flux specifies a bounding cross section in the presence of new interactions. Representing the experimental bounds in Fig. 1 with an approximate expression, and letting $\bar{y}$ be the average fraction of the neutrino's energy deposited into the shower, the cross section limit becomes

$$\sigma_{\nu N}(E) \lesssim 10^{-28}\left(\frac{10^{-18}\,{\rm cm}^{-2}{\rm s}^{-1}{\rm sr}^{-1}}{\phi_c(E)}\right)\left(\frac{10^{19}\,{\rm eV}}{\bar{y}E}\right)^{1/2}\,{\rm cm}^2\,, \tag{3}$$

where $\phi_c(E)$ is the cosmogenic neutrino flux, and the exponent $1/2$ reflects the approximate energy dependence of the Fly's Eye upper limit on the rate of horizontal air showers. For electron neutrinos and atmospheric detectors, charged current (CC) interactions correspond to $\bar{y}=1$ since all of the neutrino energy goes into "visible" energy, either in the form of an electron or of hadronic debris. In this case, using the conservative lower estimate of the cosmogenic flux in Fig. 1 yields

$$\begin{aligned}
\sigma_{\nu N}^{CC}(10^{17}\,{\rm eV}) &\lesssim 1\times 10^{-29}\,{\rm cm}^2\\
\sigma_{\nu N}^{CC}(10^{18}\,{\rm eV}) &\lesssim 8\times 10^{-30}\,{\rm cm}^2\\
\sigma_{\nu N}^{CC}(10^{19}\,{\rm eV}) &\lesssim 5\times 10^{-29}\,{\rm cm}^2\,,
\end{aligned} \tag{4}$$

and a severely degraded limit outside this energy range. The bounds Eq. (4) probe CM energies

$$\sqrt{s} \simeq 1.4\times 10^5\left(\frac{E}{10^{19}\,{\rm eV}}\right)^{1/2}\,{\rm GeV} \tag{5}$$

that are about 3 orders of magnitude beyond the electroweak scale.

In the case of neutral currents, one needs a model for computing $\bar{y}$. As an example, if $\bar{y}=0.1$ at these energies, then the best limit becomes $\sigma_{\nu N}(10^{18}\,{\rm eV}) \lesssim 2\times 10^{-29}\,{\rm cm}^2$. So as long as $\bar{y}$ doesn't depart too far from ~ 0.1, as indicated in [33], neutral currents won't change the bounds dramatically. In Section IV, we discuss the neutral current case with respect to extra dimension models.

Note that this bound does not challenge the model of Ref. [2]. That option relies on neutrinos acquiring larger, hadron-scale cross sections. As such, shower development will occur vertically, not horizontally, and our argument does not apply. We therefore specify a sub-hadron-size cross section above which our bound is inapplicable because horizontal air showers could not develop in column depths $\gtrsim 3000\,{\rm g/cm}^2$ as used by the Fly's Eye bound [12]:

$$\sigma_{\nu N}(E \gtrsim 10^{19}\,{\rm eV}) \gtrsim 10^{-27}\,{\rm cm}^2\,. \tag{6}$$

The bound derived here is independent of the type of interaction enhancing physics, stemming directly from attempts to observe deeply penetrating air showers. To summarize, Eq. (3) applies to the cosmogenic neutrino flux as long as $\sigma_{\nu N}$ does not reach hadronic levels.

We note that our allowed range, Eqs. (4) and (6), is complementary to and consistent with the interpretation of UHECRs as neutrino primaries, as considered in Ref. [5]. In this case, $\sigma_{\nu N}(E) \gtrsim 2\times 10^{-26}\,{\rm cm}^2$, which is in the larger cross section regime, Eq. (6). This is because for $\sigma_{\nu N}(E) \lesssim 10^{-28}\,\mathrm{cm}^2$, the first interaction point would have a flat distribution in column depth up to large zenith angles $\theta \lesssim 80^\circ$, whereas observed events appear to peak at column depths $\sim 450\,\mathrm{g/cm}^2$ [12]. For an individual event, a $\sim 0.2\%$ probability exists for interacting by $450\,\mathrm{g/cm}^2$ with cross sections as small as $10^{-29}\,\mathrm{cm}^2$.

We also note in this context that Goldberg and Weiler [34] have related the νN cross section at UHEs to the lower energy νN elastic amplitude in a model independent way, only assuming $3+1$ dimensional field theory. Consistency with accelerator data then requires

$$\sigma(E) \lesssim 3 \times 10^{-24} \left(\frac{E}{10^{19}\,\mathrm{eV}} \right) \mathrm{cm}^2 \,, \tag{7}$$

otherwise deviations of neutrino cross sections from the Standard Model should become visible at the electroweak scale [34].

III. FUTURE PROSPECTS FOR IMPROVEMENT

EAS detection will be pursued by several experiments under construction or in the proposal stage. As an upscaled version of the original Fly's Eye Cosmic Ray experiment, the High Resolution Fly's Eye detector [35] has recently begun operations and has preliminary results of several super-GZK events [36], but no official limit on horizontal showers thus far. The effective aperture of this instrument is $\simeq 350(1000)\,\mathrm{km}^2\,\mathrm{sr}$ at $10(100)\,\mathrm{EeV}$, on average about 6 times the Fly's Eye aperture, with a threshold around $10^{17}\,\mathrm{eV}$. This takes into account a duty cycle of about 10% which is typical for the fluorescence technique. Another project utilizing this technique is the proposed Japanese Telescope Array [37]. If approved, its effective aperture will be about 10 times that of Fly's Eye above $10^{17}\,\mathrm{eV}$. The largest project presently under construction is the Pierre Auger Giant Array Observatory [38] planned for two sites, one in Argentina and another in the USA for maximal sky coverage. Each site will have a $3000\,\mathrm{km}^2$ ground array. The southern site will have about 1600 particle detectors (separated by 1.5 km each) overlooked by four fluorescence detectors. The ground arrays will have a duty cycle of nearly 100%, leading to an effective aperture about 30 times as large as the AGASA array. The corresponding cosmic ray event rate above $10^{20}\,\mathrm{eV}$ will be 50–100 events per year. About 10% of the events will be detected by both the ground array and the fluorescence component and can be used for cross calibration and detailed EAS studies. The energy threshold will be around $10^{18}\,\mathrm{eV}$, with full sensitivity above $10^{19}\,\mathrm{eV}$.

NASA recently initiated a concept study for detecting EAS from space [30] by observing their fluorescence light from an Orbiting Wide-angle Light-collector (OWL). This would provide an increase by another factor of ~ 50 in aperture compared to the Pierre Auger Project, corresponding to a cosmic ray event rate of up to a few thousand events per year above $10^{20}\,\mathrm{eV}$. Similar concepts being proposed are the AirWatch [39] and Maximum-energy Air-Shower Satellite (MASS) [40] missions. The energy threshold of such instruments would be between 10^{19} and $10^{20}\,\mathrm{eV}$. This technique would be especially suitable for detection of almost horizontal air showers that could be caused by UHE neutrinos. As can be seen from Fig. 1, with an experiment such as OWL, the upper limit on the cross section Eq. (4) could improve by about 4 orders of magnitude. In addition, for such showers the fluorescence technique could be supplemented by techniques such as radar echo detection [41].

The study of horizontal air showers nicely complements the more traditional technique for neutrino detection in underground detectors. In these experiments, muons created in charged current reactions of neutrinos with nucleons either in water or in ice are detected via the optical Cherenkov light emitted. Due to the increased column depth (see also Fig. 3), this technique would be mostly sensitive to cross sections smaller by about a factor of 100, in a range $\sim 10^{-31} - 10^{-29}\,\mathrm{cm}^2$. For an event rate comparable to the one discussed above for ground based instruments, the effective area of underground detectors needs to be comparable and thus significantly larger than $1\,\mathrm{km}^2$, with at least some sensitivity to downgoing events. The largest pilot experiments in ice (AMANDA [42]) and in water (Lake Baikal [43] and the now defunct DUMAND [44]) are roughly $0.1\,\mathrm{km}$ in size. Next generation deep sea projects such as the ANTARES [45] and NESTOR [46] projects in the Mediterranean, and ICECUBE [47] in Antarctica will significantly improve the volume studied but may not reach the specific requirements for the tests discussed here. Also under consideration are techniques for detecting neutrino induced showers in ice or water acoustically or by their radio emission [48]. Radio pulse detection from the electromagnetic showers created by neutrino interactions in ice [49] could possibly be scaled up to an effective area of $10^4\,\mathrm{km}^2$ and may be the most promising option for probing the νN cross section with water or ice detectors. A prototype is represented by the Radio Ice Cherenkov Experiment (RICE) experiment at the South Pole [50].

The bounds we derived here can be even more constraining if the UHE neutrino fluxes are higher than the conservative estimates we used. For instance, the cosmogenic flux was computed [17] by assuming a cosmic ray spectral dependence of $N(E) \propto E^{-\gamma}$ with $\gamma = 3$, consistent with the overall cosmic ray spectrum above $\sim 10^{15}$ eV. However, at $\gtrsim 10^{19.5}$ eV, the spectrum appears to flatten down to $1 \lesssim \gamma \lesssim 2$, probably suggestive of a new cosmic ray component at these energies. Once more UHECR events are observed, the trend hinted by the AGASA data may be confirmed, giving the source of the UHECRs that generate the cosmogenic neutrinos a much harder spectral index than we assumed. The UHE cosmogenic neutrino flux computed for this flattened spectrum would yield a higher flux and a stronger bound will result. For example, assuming a rather strong source evolution and an $E^{-1.5}$ injection spectrum [14] would increase the cosmogenic flux up to 100 fold, and consequently yield a 100 fold improvement of the constraint Eq. (4). See also Ref. [51] for an estimate more optimistic by a factor 10-20 than our conservative one.

Concerning sources of primary UHE neutrinos, current estimates on the flux from the proton blazar model [52], wherein AGN accelerate protons which interact with the local thick photon field, producing pions and therefore neutrinos, could potentially improve the bound by a factor of ~ 50 at 10^{17} eV. The topological defect model shown in Fig. 1 [20] could improve the bound by a factor of ~ 75 at 10^{19} eV. These improvements would be realized if either model were to become as strongly motivated as the cosmogenic flux. Furthermore, any mechanism predicting fluxes approaching the experimental flux limits would exclude a new contribution to the νN cross section beyond the Standard Model at corresponding energies. Particularly high UHE neutrino fluxes are predicted in scenarios where the highest energy cosmic rays are produced as secondaries from Z bosons resonantly produced in interactions of these neutrino primaries with the relic neutrino background [53–55,14,20]. In general, any confirmed increase in flux over cosmogenic as currently estimated [17] would improve the bound presented here.

We also consider Markarian 421 as an example of how a point source can impact the bound. Its calculated neutrino flux is ~ 50 eV cm^{-2} s^{-1} at its peak energy of $\sim 10^{18}$ eV [56]. This is about an order of magnitude shy of the cosmogenic flux at the same energy, and therefore it will not serve to improve the bound. However, should some new evidence arise against the existence of a cosmogenic flux, point source estimates like this may become the fallback means to constrain $\sigma_{\nu N}$.

IV. ENERGY DEPENDENCE SIGNATURES AND EXTRA DIMENSION MODELS

In extra dimension scenarios, the virtual exchange of bulk gravitons (Kaluza-Klein modes) leads to extra contributions to any two-particle cross section. For CM energy such that $s \lesssim M_{4+n}^2$, where possible stringy effects are under control [57], these cross sections can be well approximated perturbatively. In contrast, for $s \gtrsim M_{4+n}^2$ model dependent string excitations can become important and in general one has to rely on extrapolations which can be guided only by general principles such as unitarity [5]. Naively, the exchange of spin 2 bulk gravitons would predict an s^2 dependence of the cross section [58,5]. However, more conservative arguments consistent with unitarity [58,5] suggest a linear growth in s, and in what follows we will assume the following cross section parameterization in terms of M_{4+n}:

$$\sigma_g \simeq \frac{4\pi s}{M_{4+n}^4} \simeq 10^{-28} \left(\frac{\mathrm{TeV}}{M_{4+n}}\right)^4 \left(\frac{E}{10^{19}\,\mathrm{eV}}\right) \mathrm{cm}^2 \,, \tag{8}$$

where the last expression applies to a neutrino of energy E hitting a nucleon at rest. It should be stressed that this cross section is only one example among a few proposed in the literature. We further note that within a string theory context, Eq. (8) can only be a good approximation for $s \lesssim M_s^2$. Above the string scale, there is currently no agreement in the literature about the behavior of σ_g with energy. Amplitudes for single states may be exponentially suppressed $\propto \exp[-s/M_s^2]$ above the string scale $M_s \sim M_{4+n}$ [57], which can be interpreted as a result of the finite spatial extension of the string states, in which case the number of states may grow exponentially [3]. This is currently unclear, and which effect dominates the total cross section may be model dependent. For example, domination by the amplitude suppression would result in $\sigma_g \lesssim 4\pi/M_s^2 \lesssim 10^{-32}\,\mathrm{cm}^2$, conservatively assuming $M_s \gtrsim 100\,\mathrm{GeV}$. Thus, an experimental detection of the signatures discussed in this section could lead to constraints on some string-inspired models of extra dimensions.

Fig. 2 shows the neutrino-nucleon cross section based on the Standard Model [32], and three curves for enhanced cross sections, given by the sum

$$\sigma_{tot} = \sigma_{SM} + \sigma_g \,. \tag{9}$$

These three curves are given for different values of the scale M_{4+n}. An increase in this mass scale brings the total cross section closer to that of the Standard Model, Eq. (1). For electron neutrinos, the Standard Model contribution is well approximated (within about 10%) by the charged current cross section Eq. (1), whereas the other flavors are harder to observe [15]. Of course, experimentally speaking, what counts for observing an interaction is the total cross section weighted by the average energy fraction transferred to the shower.

Following a format used previously in the literature [32], we can express these cross sections in a more useful manner. The interaction lengths of neutrinos through various kinds of matter can be directly compared if they are specified in terms of the corresponding interaction lengths in a particular medium. In Fig. 3, the interaction length for neutrinos is plotted in centimeters of water equivalent (cmwe), indicating the appropriate energy range necessary for detection underground and in our atmosphere. The interaction length in cmwe is computed from

$$L_{int} = \frac{1}{\sigma_{\nu N} N_A} \text{ cmwe}, \tag{10}$$

where N_A is Avogadro's number, since the density of water is 1 g/cm^3. In the particular case of extra dimension models with the cross section scaling Eq. (8), Fig. 3 shows that a neutrino becomes unlikely to create a shower below 5×10^{18} eV, since M_{4+n} can't be pushed much below about 1 TeV without its effects contradicting current experiments [4].

The total neutrino-nucleon cross section is dominated by a contribution of the form Eq. (8) at neutrino energies $E \gtrsim E_{\rm th}$, where, for $M_{4+n} \gtrsim 1$ TeV, the threshold energy can be approximated by

$$E_{\rm th} \simeq 2 \times 10^{13} \left(\frac{M_{4+n}}{\rm TeV}\right)^{6.28} \text{ eV}. \tag{11}$$

Provided that M_{4+n} is low enough, the effects of these new interactions should be observable in UHE neutrino interactions. The upper bound Eq. (3) we derived on $\sigma_{\nu N}$ by merit of current non-observation of horizontal air showers immediately implies a corresponding bound on the mass scale within the context of Eq. (8):

$$M_{4+n} \gtrsim \bar{y}^{1/8} \left(\frac{\phi_c(E)}{10^{-18}\,{\rm cm}^{-2}{\rm s}^{-1}{\rm sr}^{-1}}\right)^{1/4} \left(\frac{E}{10^{19}\,{\rm eV}}\right)^{3/8} {\rm TeV}\,, \tag{12}$$

where $\bar{y}$ enters because the bound derives from observational limits at various shower energies. Since M_{4+n} is a new constant of nature (that is, the value of M_{4+n} doesn't actually scale with E), Eq. (12) should be evaluated at the energy which generates the best limit. The strongest bound that can emerge from this formula with the lower cosmogenic curve in Fig. 1 occurs at $E = 10^{18}$ eV and $\bar{y} = 1$, giving $M_{4+n} \gtrsim 1.2\,$TeV. But for the exchange of Kaluza-Klein gravitons considered here, we can give a very rough estimate $\bar{y} \sim 0.1$ by noting similarity to massive boson exchange within the Standard Model [60]. In this case the best bound is $M_{4+n}(\bar{y} = 0.1) \gtrsim 0.9\,$TeV.

The bound Eq. (12) is consistent with the best current laboratory bound, $M_{4+n} \gtrsim 1.26$ TeV, which is from Bhabha scattering at LEP2 [61]. The fact that laboratory bounds are in the TeV range can be understood from Fig. 2 which shows that for smaller mass scales σ_{tot} would already be dominated by σ_g at electroweak scales. Eq. (12) can be improved beyond this level through more data from the continued search for horizontal showers and independently by any increase in the theoretically expected UHE neutrino flux beyond the conservative cosmogenic flux estimate shown in Fig. 1. We stress again that Eq. (12) derives from the more general bound Eq. (3) valid for CM energies about 3 orders of magnitude above the electroweak scale if $\sigma_{\nu N} \propto s$ in this energy range, and is thus complementary to laboratory bounds.

It is interesting to note that Eq. (12) does not depend on the number n of extra dimensions, assuming the cross section in Eq. (8). This is in contrast to some other astrophysical bounds which depend on the emission of gravitons causing energy loss in stellar environments [62,63]. The best lower limit of this sort comes from limiting the emission of bulk gravitons into extra dimensions in the hot core of supernova 1987A [62]. In order to retain the agreement between the energy released by the supernova, as measured in neutrinos, and theoretical predictions, the energy flow of gravitons into extra dimensions must be bounded. The strongest contribution to graviton emission comes from nucleon-nucleon bremsstrahlung [62]. The resulting constraints read $M_6 \gtrsim 50\,$TeV, $M_7 \gtrsim 4\,$TeV, and $M_8 \gtrsim 1\,$TeV, for $n = 2, 3, 4$, respectively, and, therefore, $n \geq 4$ is required if neutrino primaries are to serve as a candidate for the UHECR events observed above $10^{20}\,$eV (but see [67]). We calculated this bound for the case $n = 7$ extra dimensions, as motivated by superstring theory. We find the lower bound drops to $M_{11} \gtrsim 0.05\,$TeV, so for higher numbers of extra dimensions, the cosmic ray bound derived here could be stronger than type II supernova bounds, if the scaling assumed in Eq. (8) were widely accepted.

This assumes that all extra dimensions have the same size given by

$$r_n \simeq M_{4+n}^{-1} \left(\frac{M_{\rm Pl}}{M_{4+n}}\right)^{2/n} \simeq 2 \times 10^{-17} \left(\frac{\rm TeV}{M_{4+n}}\right) \left(\frac{M_{\rm Pl}}{M_{4+n}}\right)^{2/n} {\rm cm}\,, \tag{13}$$

where $M_{\rm Pl}$ denotes the Planck mass. The SN1987A bounds on M_{4+n} mentioned above translate into the corresponding upper bounds $r_6 \lesssim 3\times10^{-4}\,$mm, $r_7 \lesssim 4\times10^{-7}\,$mm, and $r_8 \lesssim 2\times10^{-8}\,$mm. With the cosmogenic bound and 7 extra dimensions, we have $r_{11} \lesssim 6\times10^{-12}\,$mm.

A specific signature of the linear scaling with energy of the cross section Eq. (8) consists of the existence, for a given zenith angle θ, of two critical energies. First, there is an energy $E_1(\theta)$ below which the first interaction point has a flat distribution in column depth, whereas above which this distribution will peak above the ground. This energy is independent of any experimental attributes of the detector, apart from its altitude.

Second, there is an energy $E_2(\theta) > E_1(\theta)$ below which a large enough part of the resulting shower lies within the sensitive volume to be detectable, and above which the event rate cuts off because primary neutrinos interact too far away from the detector to induce observable showers. This energy is specific to the detector involved.

We show the zenith angle dependence of these two critical energies in Fig. 4, where we have assumed approximate experimental parameters for the Fly's Eye experiment given in [12], at an atmospheric depth of 860 g/cm^2; both Auger sites are within a few hundred meters of this altitude as well. For simplicity, we have assumed that the Fly's Eye detects showers above 10^{19} eV, provided that they initiate and touch ground within 20 km of the detector. Following [2], we assume a standard exponential atmosphere of scale height 7.6 km, and a spherical (rather than planar) Earth.

These zenith angle plots can be expressed in terms of cross sections instead of energies, and be generalized to other energy dependences. They have two purposes. First, upon adding UHECR data on such a plot, a resemblance to the arcing curves shown would give strong support to the respective energy scaling. Second, in the specific case of Eq. (8), owing to the 4th power dependence on the quantum gravity scale M_{4+n}, Fig. 4 serves as a good means for determining this mass scale by comparing the data curves to those predicted. Of course, this will only work in directions in which the neutrino-induced EAS rate is not dominated by the ordinary UHECR induced rate. This restriction may necessitate additional discriminatory information such as the shower depth at maximum. Furthermore, in a detailed analysis the observed shower energy has to be corrected by the distribution of fractional energy transfer y in order to obtain the primary neutrino energy.

This approach requires more data, which will likely come from the HiRes Fly's Eye and the Pierre Auger observatories. Because these facilities can see nitrogen fluorescence shower trails in the sky, they are better equipped to see horizontal showers than ground arrays alone. In addition, they utilize more than one "eye" so their angular resolution is $\Delta\theta \lesssim 2^\circ$ [64]. On the other hand, these detectors are biased toward horizontal events because of their long track lengths, and this bias will need to be treated properly in order to interpret the zenith angle data.

As mentioned above, "traditional" neutrino telescopes based on water and ice as detector medium could be used similarly to probe a range of smaller νN cross sections $\sim 10^{-31} - 10^{-29}\,\mathrm{cm}^2$, if their size significantly exceeds 1 km and if they are sensitive to at least a limited range of zenith angles $\theta < 90^\circ$, i.e. above the horizon. For example, the ratio of upgoing to downgoing events in the $10 - 100\,$PeV range could be a measure of the absolute νN cross section at these energies [5]. However, the detection method and flavor sensitivity are different in this case.

It is worth considering whether enhanced cross sections in extra dimensions significantly raise detector backgrounds. Here, the most important background for UHE neutrino detection is from muons created in decays of collision products of UHECR interactions in the atmosphere (and to a lesser degree, from tau lepton decays). The muon range in centimeters of water equivalent, with only Standard Model interactions, is approximately [65]

$$L_\mu^{SM} \simeq 2.5\times10^5 \ln\left(\frac{2E}{\rm TeV}+1\right)\,\mathrm{cmwe}\,. \tag{14}$$

Therefore, the UHE muon range is larger than the depth of the atmosphere (see Fig. 3). But with $M_{4+n} \sim$ TeV extra dimension models, the muon acquires a cross section given by Eq. (8), and therefore has the same interaction rate in the atmosphere as the neutrinos we wish to detect. However, even if we conservatively estimate the atmospheric muon flux [14], it is more than 350 times smaller than the cosmogenic neutrino flux at $E = 10^{18}$ eV, and more than 10^3 times smaller for $E \geq 10^{19}$ eV. So with extra dimensions, atmospheric muons may lead to interesting signatures in atmospheric and underground detectors, but they do not hinder cosmic UHE neutrino shower identification.

UHECRs and neutrinos together with other astrophysical and cosmological constraints thus provide an interesting testing ground for new interactions beyond the Standard Model, as suggested for example in scenarios involving additional large compact dimensions. In the context of these scenarios we mention that Newton's law of gravity is expected to be modified at distances smaller than the length scale given by Eq. (13). Indeed, there are laboratory experiments measuring the gravitational interaction at smaller distances than currently measured (for a recent review of such experiments see Ref. [66]), which also probe these theories. Thus, future UHECR experiments and gravitational experiments in the laboratory together have the potential to provide rather strong tests of these theories. These tests complement constraints from collider experiments [6].

V. CONCLUSIONS

A direct measurement approach has been presented here for bounding the extent to which UHE neutrinos acquire greater interactions with matter than the Standard Model prescribes. By looking for nearly horizontal air showers, we can limit the UHE neutrino flux striking Earth. When this is combined with a natural source of cosmogenic UHE neutrinos, a model-independent upper limit on the cross section results. Air shower experiments can also test for the energy dependence of cross sections in the range $\sim 10^{-29} - 10^{-27}\,\mathrm{cm}^2$; we have discussed the specific case of a linear energy dependence, as motivated by some large extra compact dimensions models that have enjoyed much recent attention in the community. In such scenarios, fundamental quantum gravity scales in the $1 - 10\,\mathrm{TeV}$ range can be probed. This could also be relevant for testing string theory scenarios with string scales in the TeV range, however reliable theoretical predictions for the relevant cross sections in such scenarios are not yet available.

If this method proves beneficial, knowledge will be gained about new interactions at ultra-high energies. Should the data support extra dimensions, our understanding of UHE interactions between *all* particles (not just νN) will be changed because changes in gravity affect every particle. In general, cosmic neutrinos and UHECRs will probe interactions at energies a few orders of magnitude beyond what can be achieved in terrestrial accelerators in the forseeable future.

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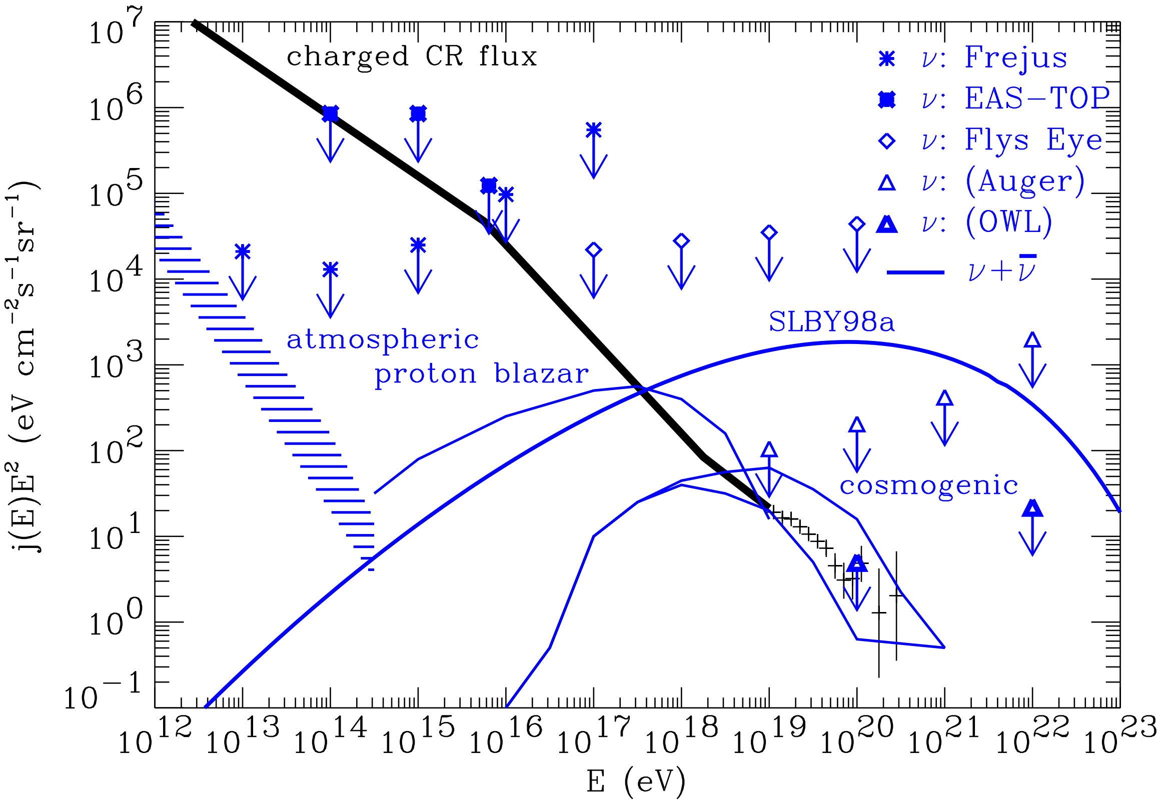

FIG. 1. Predictions for the summed differential fluxes of all neutrino flavors (solid lines) from the atmospheric background for different zenith angles [16] (hatched region marked "atmospheric"), from UHECR interactions with the CMB [17] (the two curves labeled "cosmogenic" indicating a typical uncertainty range), from proton blazars that are photon optically thick to nucleons but contribute to the diffuse $\gamma-$ray flux [18] ("proton blazar"), and for a model where UHECRs are produced by decay of particles close to the Grand Unification Scale ("SLBY98", see Ref. [20] for details). 1 sigma error bars are shown on the combined data from the Haverah Park [23], the Fly's Eye [11], and the AGASA [10] experiments above 10^{19} eV. Also shown are piecewise power law fits to the observed charged CR flux (thick solid line). Points with arrows represent approximate upper limits on the diffuse neutrino flux from the Frejus [24], the EAS-TOP [26], and the Fly's Eye [12] experiments, as indicated. The projected sensitivity for the Pierre Auger project uses the acceptance estimated in Ref. [27], and the one for the OWL concept study is based on Ref. [30], both assuming observations over a few years period. Note that this plot supposes that $\bar{y} = 1$; for $\bar{y} < 1$, data points correspond to an x-axis of observed shower energy, and the predicted flux curves correspond to an x-axis of incident primary energy. If the x-axis is to be interpreted as observed shower energy for $\bar{y} < 1$, then the predicted flux curves should be shifted to the left by a factor of $\bar{y}$ and multiplied by $\bar{y}$.

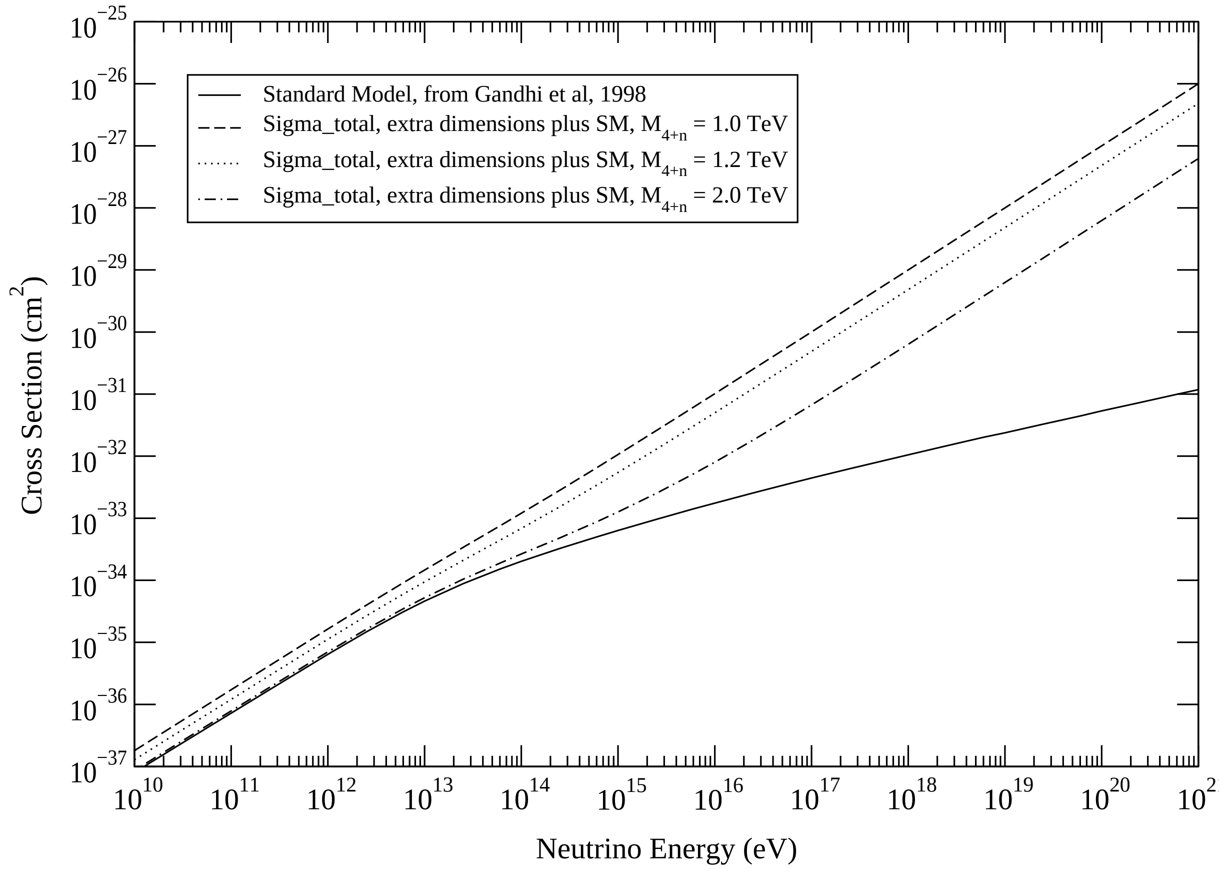

FIG. 2. Cross section for νN interaction, assuming neutrinos incident on nucleons at rest. The solid curve is for the Standard Model, charged current [32], as appropriate for electron neutrino detection (see text). Remaining curves give Standard Model plus the contribution from extra dimensions at various scales in the parameterization of Eq. (8): dashed line for $M_{4+n} = 1.0$ TeV, dotted line for $M_{4+n} = 1.2$ TeV, dot-dashed line for $M_{4+n} = 2.0$ TeV.

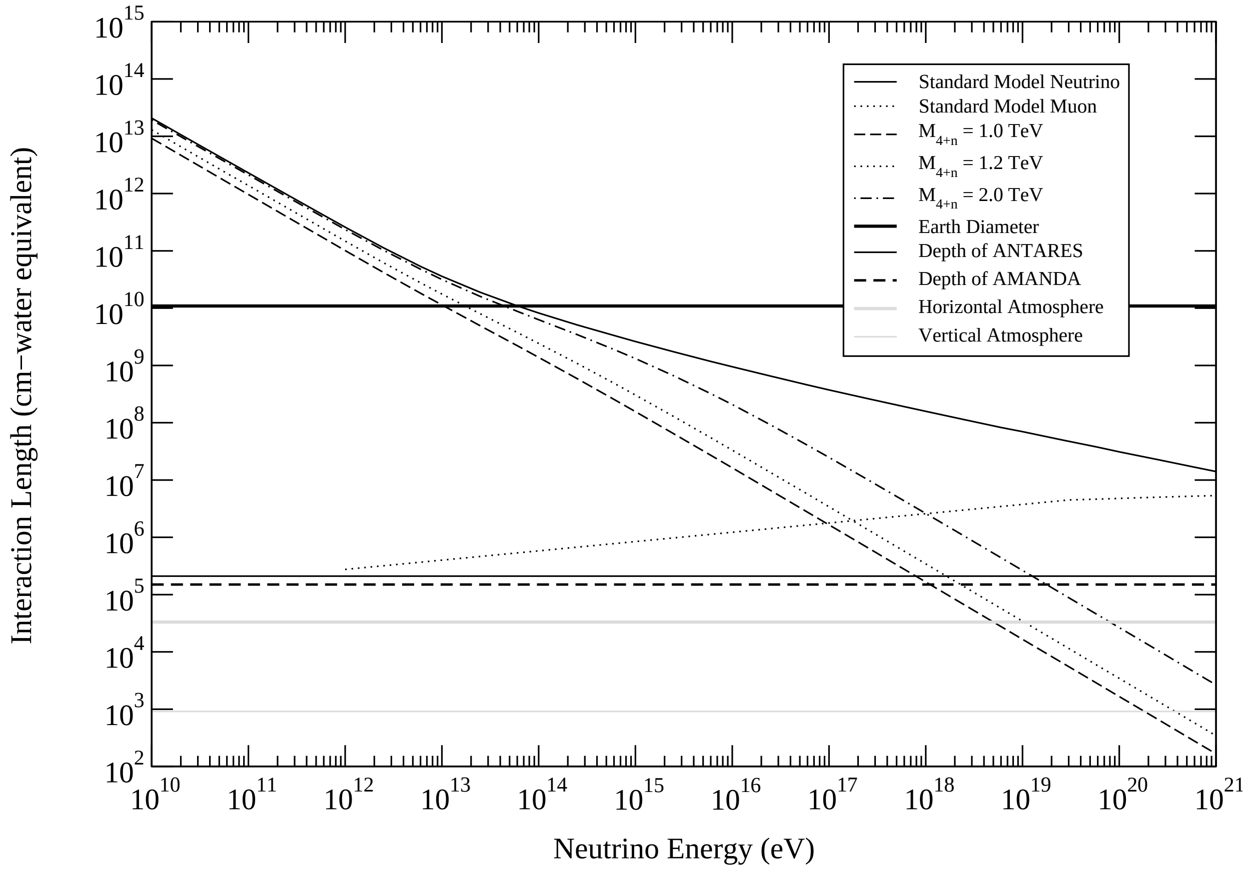

FIG. 3. Interaction length for νN interaction in water or ice, assuming neutrinos incident on nucleons at rest, similar to plots in Ref. [32]. Down-sloping diagonal lines correspond to the same cross sections as in Fig. 2. The up-sloping dotted line is the Standard Model muon range. Horizontal lines, from top to bottom, correspond to the water equivalent of the Earth along a diameter, the depth of the ANTARES and AMANDA neutrino observatories, the thickness of the atmosphere taken horizontally (along a tangent to the spherical Earth's surface), and the thickness of the atmosphere taken vertically (zero zenith angle). The Earth diameter in cmwe is computed from a parameterized Earth model [59] of Earth's internal structure, and the atmosphere is treated as exponential, as defined in the text.

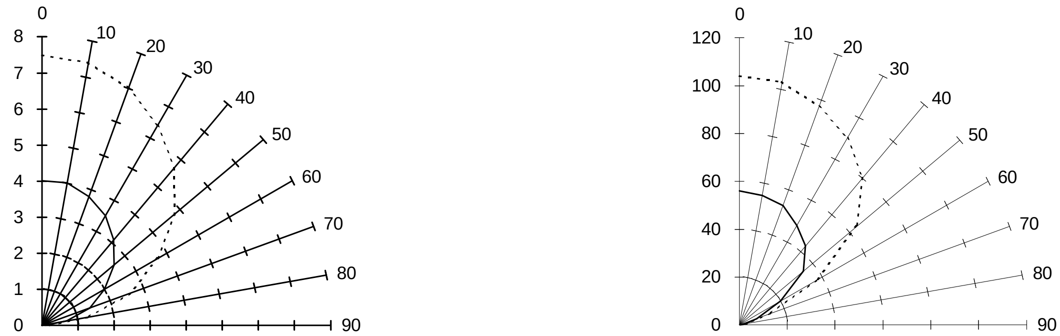

FIG. 4. The critical energies $E_1(\theta)$ (left panel) and $E_2(\theta)$ (right panel) as a function of zenith angle for neutrino-induced air showers, as discussed in the text. Energy is plotted radially in units of 10^{20} eV. The solid curve is for $M_{4+n} = 1.2$ TeV and the dashed curve is for 1.4 TeV, for comparison. These curves apply to detectors located near an atmospheric depth of 860 g/cm^2, including Fly's Eye and both Auger sites.